MODULE ONE

Numbers

MODULE ONE – NUMBERS

Contents

UNIT ONE

Understanding processes of calculations

> ***Trainer's Note:***
> *Before beginning this unit, talk to learners about some of their early experiences of learning mathematics, particularly in relation to number work. Ask questions like:*
> - *What calculation methods did they learn at school?*
> - *How were these taught?*
> - *What happened to them if they used their own methods that were different to the ones their teachers used?*
> - *Was this encouraged or discouraged?*
> - *Were the methods their teachers used always understood or easy to follow?*
> - *Did they learn "rules" to calculate by rote or with understanding?*
> - *What has been the long term effect of their learning to do a particular calculation in only one way?*
>
> *In general has their school learning experience helped them to like mathematics, explore their own ways of thinking or extended their mathematical abilities?*

In this unit you will address the following:

Unit Standard 7447

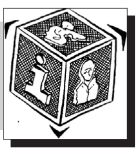
SO1:
Express and interpret a range of contexts using mathematical symbols and find applications for numerical models. (Everyday problems, numerical contexts. Numerical models include equations, expressions and terms.)

SO2:
Solve a range of everyday problems using estimation and calculations. (Rounding off and truncating, with or without calculator, combination, separation, comparison, equalization, sharing and grouping).

SO3:
Verify and justify solutions within different contexts. (Solutions derived by learners and by others).

SO4:
Perform operations on simple and complex numerical expressions. (four basic operations in all combinations; expressions involving exponents that can be calculated without a calculator; operations to be performed with and without a calculator).

To do this you will:
- explain the difference between following a method to solve mathematical problems without an understanding of why this produces the desired results ("Magic") and…;
- following a method with an understanding why this produces the desired results ("logic");
- follow, use and explain different methods for approaching tasks that require multiplication and division of whole numbers and division of fractions;
- re-evaluate rules that you might have been taught at school to approach similar tasks, explain why they work and also evaluate if there are better ways to do the same task that make more sense to you;
- think of ways that you can apply what you have learnt to your work with young children.



1. Magic or Logic?

In mathematics, there is no such thing as "magic". There is always someone who can explain why a mathematical method or process works. When many of us were at school our teachers gave us rules that we had to follow blindly to try and get the right answer, hopefully without making too many messy marks in our books!

We were told: "That's the rule, there's no other way. It always works. Don't ask why!"

DICTIONARY:

investigate – explore

In this unit you will investigate ways of working with both whole numbers and fractions that are probably new to you and unlike any methods you might have been taught when you were at school.

You will share your methods with others and see how you can sometimes use different ways to arrive at a solution to the same problem. You can then evaluate these different ways and see which one makes most sense to you. In this way you will understand that there is no "magic" involved in these calculations - every method can be logically explained.

You can then apply what you have learned to your work with young children.

In this unit we assume that you understand multiplication and division facts and you know how to use the basic operations to do calculations with numbers into the thousands. If you are not confident in these areas, you can refresh you memories by practicing examples in materials set at an ABET Level 3 or from school textbooks. (Grades 5-7)

Time needed 60 minutes

Activity 1:
The Russian Peasant Method of Multiplication

Work alone

Knowing mathematics means understanding it. When we memorise rules for moving symbols around on paper, we may be learning something but we are not learning mathematics. Learning mathematics means getting inside it and seeing how things work, how things are related to each other, and why they work the way they do.

There is an ancient method of multiplication that is still used in some parts of the world (e.g. by many people in Ethiopia). It is sometimes referred to as the "Russian Peasant" method.

We can summarise these steps as follows.

$$\begin{aligned}
&35 \times 65\\
&= (34 \times 65) + 65\\
&= (17 \times 130) + 65\\
&= (16 \times 130) + 130 + 65\\
&= (8 \times 260) + 130 + 65\\
&= (4 \times 520) + 130 + 65\\
&= (2 \times 1040) + 130 + 65\\
&= (1 \times 2080) + 130 + 65
\end{aligned}$$

Work in a group

1. Apply the "Russian Peasant" method to 49 × 35. When you have done this calculate 49 × 35 in a way you would normally use.
2. Apply the Russian peasant method to any two other numbers with two or more digits.
3. Check whether it gives the same answer as when you multiply in the way you normally use.
4. Discuss: Why do you think the ancient method of multiplication works?

Understanding the method

There is a logical explanation for why the Russian peasant method of multiplying produces the correct answer each time.

By doing the next task and following the steps in the given order, you will come to understand why and how this ancient method of multiplication works. At the same time you will learn some of the informal language that can be used to help explain each step followed.

Time needed
15 minutes

> ***Trainer's Note:***
> *The purpose of doing these practise activities here is that doubling and halving are useful strategies to use when doing multiplication and division and will be demonstrated in the next section that unpacks the Russian Peasant method of multiplication.*

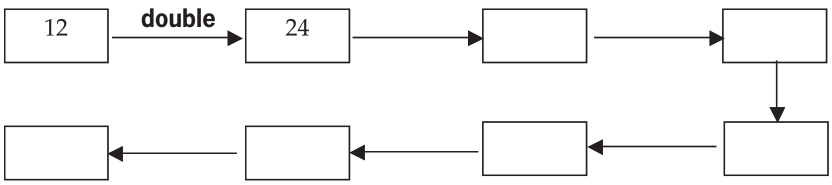
Activity 2: Practicing doubling and halving

Work alone

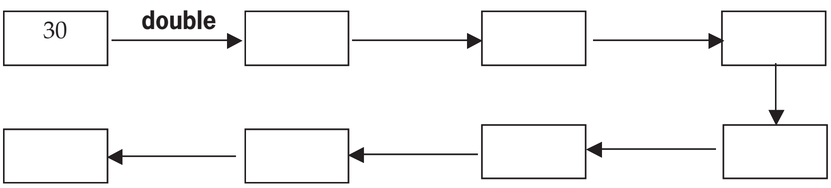
Before you go any further with this investigation let's first practice doubling and halving numbers.

1. Double these numbers to complete the chain:

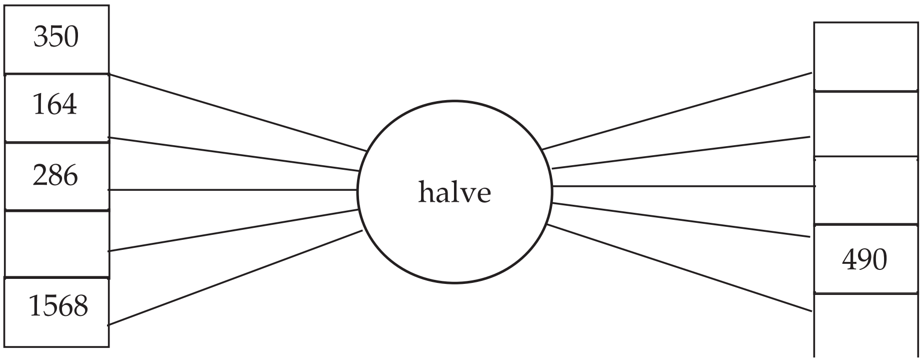
a)



b)



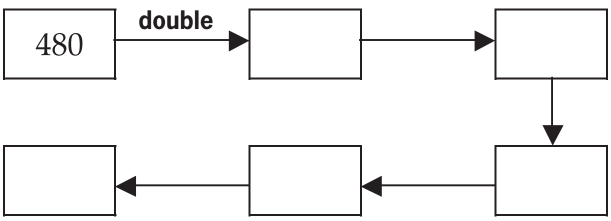
2. Now use halves to fill in the missing numbers



3. Complete these chains

a)

b)



Time needed
40 minutes

Activity 3:
What is the logic?

Work in pairs

Follow these steps to uncover the logic of the Russian Peasant multiplication method using the numbers 35 × 65.

Step 1

35 × 65 = 34 × 65 + **65**.

We will keep 65 as part of the answer and work further with 34 × 65.

Step 2

34 × 65 = half of 34 times double 65 which is 17 × 130.

17 × 130 = 16 × 130 + **130 + 65**.

We will keep 130 and 65 as part of the answer, and 65 from Step 1, and work further with 16 x 130.

Step 3

16 × 130 = 8 × 260

8 × 260 = 4 × 520

4 × 520 = 2 × 1040

2 × 1040 = 1 × 2080 = **2080**.

So the answer for 35 × 65 is **65 + 130 + 2080.**

2080 + 130 + 65 = 2275

> ***Trainer's Note:***
> *Make time for the groups to share their ideas in relation to these questions above. Also think about how you will use the approaches demonstrated by this activity when you mediate the rest of this programme with your learners. For example, encourage them to come to their own solutions and to use and value different methods to approach the same task.*

Work in pairs or a group

1. Discuss these two statements in relation to what you have learnt or experienced by doing this activity:

   Statement 1
   "Mathematical logic is there for us to discover in the same way they discover that fire will burn or that a sharp knife can cut."

   Statement 2
   "Mathematical logic is not "discovered", but developed as you work with it over a long period of time"

2. Now answer these questions:
   a. Is there really such a thing as mathematical magic? Explain why or why not?
   b. What have you learnt about mathematical logic from doing this activity?
   c. How can you use the principles of what you learnt from this investigation when you approach other number tasks?

2. Exploring division methods

Do you remember learning the long division method at school? In this activity you are going to look at different ways of solving a "long division" problem. By the end, the standard method should make more sense to you and in addition you will have found that there are other ways to solve the same problem that make more sense than the traditional method.

> ***Trainer's Note:***
> *The purpose of this activity is to find out about learners' understanding of division and about the methods they would be familiar with. It is likely that most of them will have learnt the traditional long division algorithm, but in a mechanical way, rather than with understanding. The next section will demonstrate some of the many different ways to approach a long division problem, and give learners a range of options that might make more sense than the traditional method they probably learnt by rote when they were at school.*

Activity 4:
Exploring division methods

Work alone

1. Use any methods you know to solve these division problems on your own. Only use a calculator when you are finished as a checking tool, rather than to find the answer. You will share your methods later on.

   a. The Bantwana Bami ECD centre needs 880 litres of paint to repaint their classrooms. They can buy paint in 65 litre drums directly from a factory. How many drums will they need? Show all your calculations to explain your method.

   b. At a recent Governing Body meeting of the centre it was decided to share the profit from the sales of the vegetable garden between the volunteers that worked there. The profit was R6 478. There were 37 volunteers. How much should each volunteer get if they all get an equal share of the profit?

Work in a group

2. Look at the ways some different learners approached this problem.

   Moshanke's way:

3700	R100	each
370	R10	each
740	R20	each
4810		
740	R20	each
740	R20	each
6290		
74	R2	each
6364		
74	R2	each
6438		
37	R1	each
Total: 6475	**R175 each**	**and R3 left**

Gertrude's way:

Maybe each one will get at least R100, that will be 37 × 100 = R3 700. I can see there is much more money, but not enough for R200 because that will be R7 400. So each one will get at least R150, that will be R3 700 + R1 850 = R5 550.
Then there will be R6 478 – R5 550 = R928 left. 370 + 370 ⟶ 740 + 370 = 1 130.
So each one can get another R20.
Then there is R928 – R740 = R188. So each one can get another R2, that is R74,
And another R2, that is R148, and another rand, that is R148 + R37 = R185.
So there is R3 left and each one gets R150 + R20 + R2 + R2 + R1 = R175.

Thabo's Way:

	Profit =	6478	
100	× 37 =	3700	
	R100 and	2778	remain
50	× 37 =	1850	
	R150 and	928	remain
20	× 37 =	740	
	R170 and	188	remain
4	× 37 =	148	
	R174 and	40	remain
1	× 37 =	37	
R175	And	R3	remain

Tabitha's Way:

100	3700	
100	3700	
	7400	too much
50	1850	
150	5550	
10	370	
10	370	
10	370	
180	6660	too much
170	6290	
5	185	
175	6475	
R175 and R3 left		

Khanyi's Way:

175	
	1
	4
	20
	50
	100
37	6478
	3700
	2778
	1850
	928
	740
	188
	148
	40
	37
	3

Nathi's way:

Maybe each one can get R200.37 × R200 = R7 400, the money is short. I will try R150. There is much money left. I will try R160. I will try R180. I see that is too much. I will try R175. That is right. There is R3 left.

> ***Trainer's Note:***
> *Before moving ahead, have a discussion with your learners about all these different methods. Make sure they can follow each way and explain all the steps involved. Have them evaluate which in their view is the easiest method or the one that makes most sense to them.*

Sydney's way:

$$
\begin{array}{r|l}
 & \overline{175} \\
37 & 6478 \\
 & \underline{3700} \\
 & 2778 \\
 & \underline{2590} \\
 & 188 \\
 & \underline{185} \\
 & 3
\end{array}
$$

Each volunteer gets R175 and 3 cents = R175,03

Did you know that large tracts of land are measured in hectares? One hectare is the same as 10 000 square metres.

> ***Trainer's Note:***
> *This may be a good time for you to explain to learners that we have not provided the answers to every single problem in this manual. This is because we want to encourage learners to understand the processes, so that they can find the answers independently. We want to encourage an approach of working things out together, finding and discussing mistakes, and learning from mistakes. Sometimes answers are given so that learners can check if they are on the right track. You as the trainer will need to do preparation work, sometimes working out solutions before the training. This will enable you to give clearer guidance when mistakes are identified.*

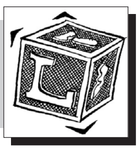
Activity 5:
Comparing long division methods

Work alone

1. Use a method of your own to do the same calculation. Compare this method to the ones used by the people in the examples above. How is it the same? How is it different?
2. The following tasks will help you to find some of the differences and similarities between the methods shown above.
   a. Use Moshanke's or Tabitha's method to solve the following problem:
      8036ha of land is to be divided into 28 farms of equal size. How big should each farm be?
   b. Use Nathi's method to solve the following problem:
      47 parents agree to contribute equally to raise R15 416 to pay an outstanding electricity bill, How much does each parent have to contribute?
   c. Compare Moshanke's and Tabitha's methods. Write brief notes to describe the similarities and the differences.
   d. Compare Sydney and Khanyi's methods. Write brief notes to describe the similarities and the differences.

Work in pairs

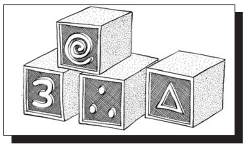
3. Compare your responses to questions a, b, c and d.



What have you learned?

Now you can see that there are many different ways to solve the same problem whether it is multiplication, division, or even addition and subtraction. The ways we were taught to do things at school often did not make sense and we just had to follow rules and not ask questions!

An investigative approach to teaching and learning mathematics says that not all people think in the same way and therefore when they approach a mathematical task they may naturally want to do it differently to the way they have been taught. This encourages learners to think and to logically explain their methods and how they arrived at their answers.



Linking your work with children

When you work with numbers, you can encourage children wherever possible to ask questions and use their own language to describe their ways of thinking. By doing this you will be helping to promote a generation of young learners who are independent, critical, expressive and self–reflective, and who know how to think and speak mathematically!

Young learners enjoy number challenges especially in the form of stories. You can use simple numbers and build a story around them. Depending on the ages of your learners you can use the situation to develop different number concepts and ways of thinking.

Here is an example to try out with two different age groups of learners. These are roughly determined. The dialogue we have constructed is only a suggestion of how to mediate the activity. You can adapt or translate it freely. But try to keep the sense and direction of the questions and tasks we suggest.

Ages 3-4

Give me one more ball to put in my box. How many are there?

    Four!

Good! Let's share them between the two of us… one for you one for me etc

So we got 2 each by sharing these 4 between us, do you agree?

    Yeh! I got 2 and you got 2 also!

So we can say that 4 makes two twos, or 2 and 2 make 4 altogether.

Did you know we call things that come in 2s pairs, like pairs of socks?

..or pairs of gloves!

Now let's try sharing more balls between us. Here let's add some more to the pile.

Learners do this and they explore sharing larger amounts where there is a remainder. They discuss ideas on what they should do with the one(s) left over after the others have been shared out equally.

Ages 4-5

Teacher: Let's pretend these piles are sandwiches.

Let's make four piles of sandwiches, with three sandwiches in each pile.

(The children do this under guidance)

Right! Now find a quick way to count them to find out how many sandwiches there are altogether!

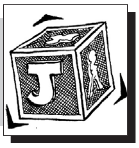
Learner: Here's one pile with 3 sandwiches. 2 piles is double that so that's 6 sandwiches.

Teacher: And if you double that?

Learner: Double 2 is 4 … So in 4 piles there will be 12 altogether!

Teacher: So if you take 12 sandwiches and share them among 4 of us, how many will we all get…?

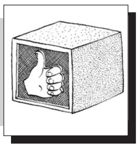
Children continue with the investigation, exploring links between language of grouping and sharing.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about magic or logic?
b. Write down one or two questions that you still have about multiplication and division.
c. How will you use what you learned about magic and logic in your everyday life and work?



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Follow a method to solve mathematical problems without an understanding of why this produces the desired results (“Magic”) and…				
2. Follow a method with an understanding of why this produces the desired results (“logic”, rational understanding)				
3. Follow, use and explain different methods for approaching tasks that require multiplication and division of whole numbers				
4. Re-evaluate rules that have been taught at school to approach similar tasks, explain why they work and also evaluate if there are better ways to do the same task that make more sense to you				

UNIT TWO

Exploring number bases

DICTIONARY:

convert – change
value – how much

In this unit you will address the following:

Unit Standard 7449
SO3:
Critically analyze the use of mathematics in social relations. (Social differentiation: gender, social mobility, race; historical and possible future contexts, e.g. employment equity; apartheid policies).

Unit Standard 7447
SO4:
Perform operations on simple and complex numerical expressions. (four basic operations in all combinations; expressions involving exponents that can be calculated without a calculator; operations to be performed with and without a calculator).

SO5:
Describe and compare counting systems from different cultures. (own, African culture other than own, one other).

SO6:
Critically analyze the development of the base ten number system. (Place value, role of 0 in our number system, patterned nature of whole numbers, history and contestations).

To do this you will:
- explore the Mayan base 20 number system;
- count and calculate in the Mayan system and;
- convert between base 20 and base 10.

Stop and Think
If you were going to invent mathematics from scratch what aspects of your maths would have to be the same as the maths that is used around the world today? What aspects could be different? Take a few minutes to discuss this question with your colleagues.

1. Base Ten

Every civilization has developed mathematics to meet its needs. Many cultures developed mathematics using base 10, probably because you have 10 fingers. The system you use today was invented in India. Later Arab scholars extended the system to include decimal fractions. When you write a number like 1984 you know that each digit has a certain value that depends not only on the digit itself but also on the place it occupies. You talk about the units place, the tens place, the hundreds place, the thousands place etc. For whole numbers you start with the units place, and you add as many places to the left as you need, each place being ten times more than the one before. So, another way of saying the thousands place would be to call it the 103 place, for the hundreds place you could say the 102 place, for the tens place you could say 101 and for the units place you could say 100 (any number to the zeroeth power is 1).

If you want to work with fractions you can add more places to the right, each place being ten times smaller than the one before.

This system is very powerful. You need only 10 symbols – 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 to write very big numbers or very small numbers. You can add, subtract, multiply and divide, as long as you remember that the value of each position is ten times greater than the place to its right. This is the secret behind the regrouping you do when you carry in addition or borrow in subtraction.

$10^3=1000$	$10^2=100$	$10^1=10$	$10^0=1$
1	9	8	4

$1 \times 10^3 = 1\ 000$
$9 \times 10^2 = 900$
$8 \times 10^1 = 80$
$4 \times 10^0 = 4$

2. Other Bases

Some numerical systems use base 10, but don't use place value. Roman numerals, for example, were used in Europe until about 1200AD. You still use Roman numerals today for some dates. If you see MCMLXXXIV on a building it was built in 1984. The Roman system does not use place value ,so it is not very useful for calculating.

Other cultures used different bases. Some used base 5, perhaps because you have 5 fingers on each hand. People who used the spaces between the fingers to count, instead of the fingers themselves, used base 8. Some cultures used base 12. The Sumerians used base 60. You have inherited the way you divide an hour into 60 minutes and 60 seconds, and degrees into 60 minutes and 60 seconds from the Sumerians.

Almost any number can be used as a base. But our system is good for calculating because it uses place value.

The binary, or base 2, system was invented in ancient China. It became very important in the last century with the development of computers because it only needs two symbols – a 0 and a 1. Computers are made up of transistors – electronic switches that have just two states. If you call one state 0 and the other state 1 you can build a computer from transistors that can carry out arithmetic functions.

In base 2, the value of each place goes up by a factor of 2 each time you move to the left, and down by a factor of two each time you move to the right.
Let's count to ten in binary: 1, 10, 11, 100, 101, 110, 111, 1 000, 1 001, and 1 010.

Time needed
30 minutes

DICTIONARY:

accurate – correct

Activity 1:
Working with a binary number system

Work in pairs

1. Write the number 11 using the binary system.
2. Read this large number to understand how it is written using the binary system

$2^{10}=1024$	$2^9=512$	$2^8=256$	$2^7=128$	$2^6=64$	$2^5=32$	$2^4=16$	$2^3=8$	$2^2=4$	$2^1=2$	$2^0=1$
1	1	1	1	1	0	0	0	0	0	0

3. What is the base ten equivalent of the binary number 11 111 000 000?

3. The Mayan System

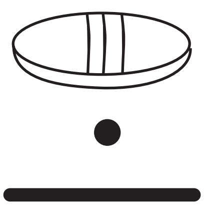
Long before the Europeans knew about America or the zero, a high culture flourished on that continent. Descendents of this culture live today in Guatemala, Mexico and Belize.

The ancient Maya discovered zero. This allowed them to build a sophisticated mathematical system, which in turn allowed them to understand Astronomy and develop a highly accurate calendar.

They chose 20 as the base of their number system. People, after all, have 20 "digits" - five fingers on each hand, five toes on each foot.

Just as the Chinese used beads on an abacus to calculate, the Mayans used beans, sticks and shells to carry out arithmetic in their system.

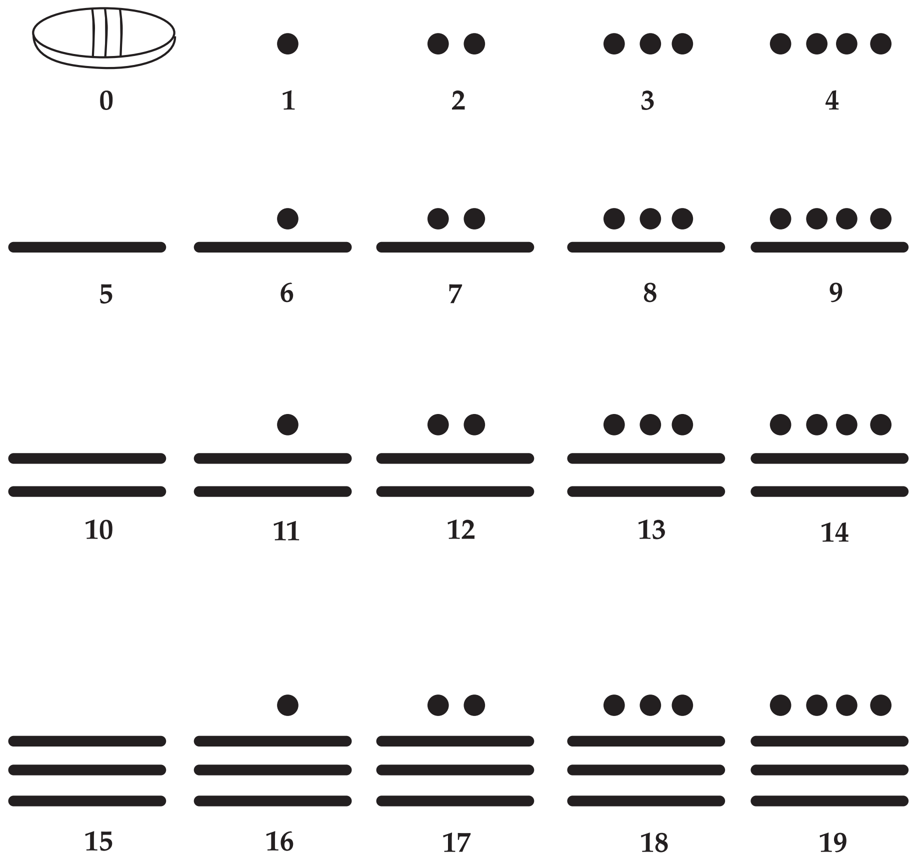
A system needs a different symbol for each number in its base, so the Mayans needed 20 in all, including the zero. For the zero they used a shell to calculate and a picture of a shell when they were writing. But they realized they could build up the numbers from 1 to 19 by using a bean (or a dot in writing) to represent a one, and a stick (or bar in writing) to represent a 5.



Zero is represented by a shell or a picture of a shell.

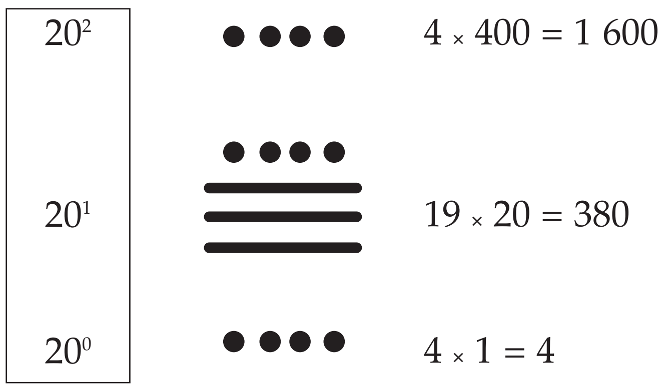
One is represented by a bean or a dot
Five is represented by a stick or a bar.

Here's what the Mayan numbers from 0 to 19 look like.



Instead of arranging the places as you do, left to right, the Mayans went vertically. They started with the units place (which in this case was 20^0). Then they went up as many places as they needed. The second level was the twenties place (20^1). The third level was 400s (20^2). Then came 8 000s (20^3), etc. So they got to very large numbers very quickly.

Here's what 1984 looks like converted to Mayan numbers.

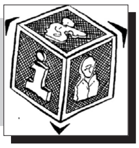


Trainer's Note:
This activity will help learners to understand the meaning of place value. The most important idea is that ANY number can serve as a base. What makes a system powerful is not the choice of base, but the use of place value. Encourage learners to use the toothpicks and beans rather than doing mental maths. It will prove much easier for them.

Activity 2:
Using the Mayan Counting System

Work alone

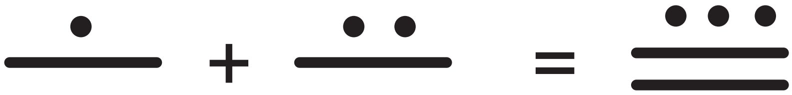
1. Form the following numbers in the Mayan system. Use toothpicks for 5 (bars) and buttons or beans for 1 (dots) and a counter for zero.
   a) 421 b) 2 105 c) 401



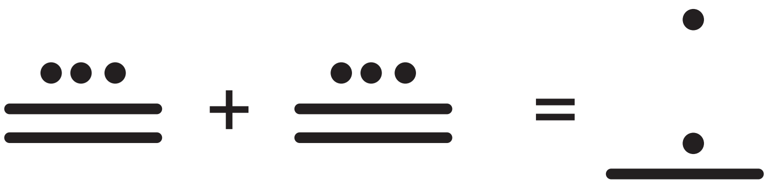
4. Addition in the Mayan system

Addition is easy, too. Add the dots. If you have five dots, replace them with a bar. Add the bars. If you have four bars, replace them with a dot at the next higher level. Look at these examples:

Example 1
6 + 7 = 13



Example 2
13 + 13 = 26 Here you have to convert four bars to a dot at the level above, and five dots are converted into one bar.



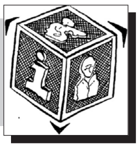
Time needed
20 minutes

Activity 3:
Adding using the Mayan system

Work alone

1. Use toothpicks for bars and beans or buttons for dots to work out the following sums.

   a) 10 + 11 b) 19 + 2 c) 345 + 63 d) 499 + 21

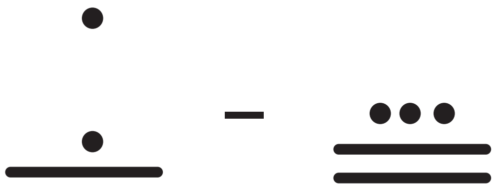


Trainer's Note:
Learners will get great satisfaction when they are able to master addition and subtraction in Mayan Math. If you find they are having difficulty, tell them NOT to think about the value in base TEN. Encourage them to physically combine all the symbols in the units place. First, if they have five beans they should exchange them for 1 stick. When they have fewer than five beans, they should then consider the sticks. If they have four sticks they should exchange them for one bean in the next level up. Zeros are used to identify a level without any beans or sticks.
Once they have mastered the method they will be able to calculate quickly and easily. You might suggest that they invent sums for one another.

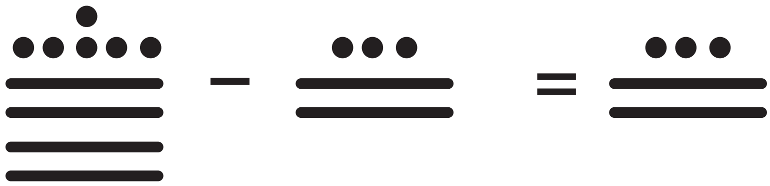
5. Subtraction in the Mayan system

Subtraction works the same way. If you borrow you have to convert a dot into four bars at the level below.

Let's subtract 13 from 26:



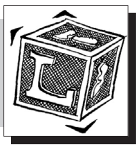
We need 2 bars and three dots in the units place. You need to do two things. You will convert the dot in the 20s place to four bars, each representing 5, in the units place. You will convert one bar to five dots. Now we're ready to take away two bars and three dots. What's left? Two bars $2 \times 5 = 10$ plus three dots $10 + 3 = 13$. $26 - 13 = 13$.



Activity 4:
Subtracting using the Mayan system

1. Use your toothpicks and buttons, work out these differences.

   a) 5 – 1 b) 20 – 1 c) 400 – 1 d) 421 – 105



What have you learned?

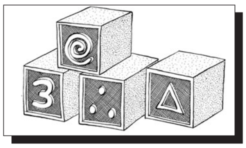
There have been many different mathematical systems developed by people around the world. Some use base 10, others use different numbers. Only a few civilizations – some historians believe only two, the ancient Hindus in India and the ancient Mayans in Central America – discovered the place value system, which depends upon the use of zero to show that a place exists even if it is empty for the moment.

Any number can serve as a good base. The Mayans achieved great strides in science with their base 20 system. Computers calculate at the speed of light using base 2. Most of us do most of our mathematics in base 10. Place value makes a system more powerful.

We have also learned how to convert numbers, and to add and subtract in the base 20 system used by the ancient Mayan civilizations.

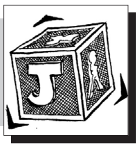
DICTIONARY:

quantity – amount



Linking your learning with your ECD work

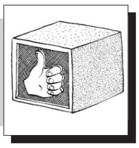
Children need to work with concrete objects to get a good number sense. Reciting is not enough. They need to understand that each number represents a quantity. Play counting games with buttons and sticks. Play exchange games like 5 buttons for one stick. This will help children develop the understanding of number and lay the foundation for ideas like place value, borrowing, and regrouping.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about other number systems?
b. Write down one or two questions that you still have about other number systems.
c. How will you use what you learned about other number systems in your everyday life and work?

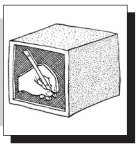


Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Explore the Mayan base 20 number system				
2. Count and calculate in the Mayan system and				
3. Convert between base 20 and base 10.				



Assignment 1:

Write the year you were born in Mayan numerals. Then write your age. Add the two together, regrouping and carrying where you need to. If you have already had a birthday this year, add 1, regrouping if necessary.

Label the units place (200), the 20’s place (201), and the 400’s place (202). Convert the answer to decimals. What is the sum?

UNIT THREE

Working With Powers

DICTIONARY:

notation – way of writing

In this unit you will address the following:

Unit Standard 7447
SO4:
Perform operations on simple and complex numerical expressions. (four basic operations in all combinations; expressions involving exponents that can be calculated without a calculator; operations to be performed with and without a calculator).

SO6:
Critically analyze the development of the base ten number system. (Place value, role of 0 in our number system, patterned nature of whole numbers, history and contestations).

To do this you will:
- use power notation to write large and small numbers in short form;
- explain what the terms power, base and exponent mean;
- interpret a range of different numbers expressed in powers, not only those written to the power of 10;
- identify and interpret examples in the real world where power notation is used as a way of expressing larger or smaller numbers;
- do your own research to find large measurements that can be expressed using scientific notation.

Trainer's Note:
Emphasize the way we use symbols and notation in mathematics in order to condense large amounts of information so that we can work more efficiently.

1. Powers of 10

You know well now that our number system follows a base of 10. The place of the digits in a number gives the digit its value. Each place, working from right to left, is ten times bigger than the place before it. As numbers can get very big, mathematicians have come up with a way of writing these numbers in short form using powers. You will now look at what this means and how it works for whole numbers.

**Time needed
20 minutes**

Activity 1:
Powers of 10.

Work alone

1. Write quick answers to the following:
   10×10
   $10 \times 10 \times 10$
   $10 \times 10 \times 10 \times 10$
   $10 \times 10 \times 10 \times 10 \times 10$
   $10 \times 10 \times 10 \times 10 \times 10 \times 10$
   $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

2. Discuss what you notice about the number of zeroes in the answer and the number of tens that are multiplied together.

Trainer's Note:
Give learners more practise in using the different ways of expressing power notation values and to help them to distinguish between the base and the exponent value in each case.

What have you learned?

You probably noticed that there is the same number of zeroes in the answer as the number of tens in the multiplication. We can write all of these numbers in short form like this:

One hundred = $10^2 = 10 \times 10 = 100$
One thousand = $10^3 = 10 \times 10 \times 10$
Ten thousand = $10000 = 10^4 = 10 \times 10 \times 10 \times 10$
This is called power notation. You can say "10 to the power 2.", or "10 to the power 3", or "10 to the power 4". You call this writing the number as a power of 10. 10 is called the base. ^{2} is called an exponent.

Activity 2:
Working with powers of 10

Time needed 25 minutes

Work alone

1. Write out in full how many metres is each of the following distances.
   a. The distance across a certain city is 10^4 metres
   b. The distance across the centre of the earth is 10^7 metres
   c. The distance across the sun is 10^9 metres

Work in pairs

2. Discuss how you would convert each measurement above to kilometres, and then write each measurement above in kilometres.

What have you learned?

In the examples above you worked with numbers up to 10^9. But you can go on and on like this to get numbers in the hundreds and thousand of billions that you can write in short form using powers. Perhaps you even experimented with numbers higher than that. Take for example the googol. A googol is bigger than a million, million. You write it like this 10^{100} . The name was invented by the son of an American Mathematician Edward Kasner when his father asked him to make up a name for a very large number!

When you use power notation the base does not have to be 10. Look at the example of a cubic number like $2 \times 2 \times 2 = 2^3$. You can say the number 2^3 is a power; where 2 is the base and ^{3} is the exponent. In this case the answer is only 8. But 2^3 is quicker to write than $2 \times 2 \times 2$.

You know that sometimes in mathematics letters are used for numbers that you do not know. You can do the same in power notation. Look at these numbers:

3^x and x^5

3^x is read as "3 to the power x" and x^5 is read as "x to the power of five" or "x to the 5th power".

Time needed
60 minutes

Activity 3:
Practice writing power notation

Work alone

1. Write these numbers as powers:
   a) 3×3 b) $5 \times 5 \times 5$ c) $y \times y \times y$ d) $a \times a$
   f) $6 \times 6 \times 6$ g) $1\,000 \times 1\,000$ h) $l \times l \times l$ e) $2{,}5 \times 2{,}5$
2. Write these numbers in words and in expanded form.
   e.g. a^4 is " a to the 4th power" or $a \times a \times a \times a$

   a) m^3 b) x^7 c) c^3 d) d^2 e) 5^6

3. Calculate the value of:

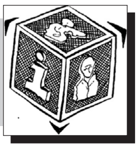
   a) 42 b) 33 c) 54 d) 63 e) 107 f) 25

4. Write these as exponents and then in expanded form

   a) y to the power of 4 b) y to the five c) p squared d) n to the power of 7

Work in pairs

5. Compare your answers with a partner's. Make any corrections you need to.



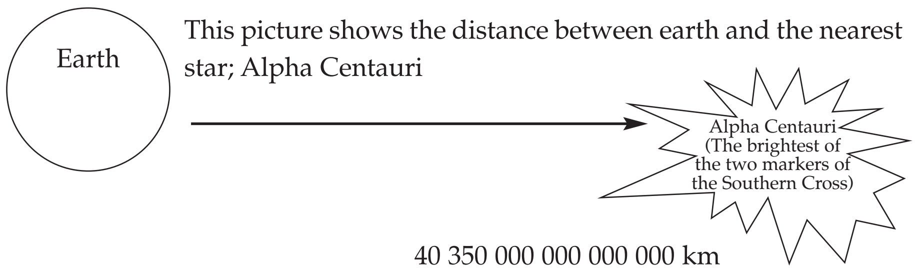
2. Using scientific notation for large numbers

Scientists often deal with very large and very small numbers. They use scientific notation to write these numbers.

Trainer's Note:
Make sure learners understand these steps before moving ahead. Give more practice using similar examples if necessary.

When you write a number in scientific notation you write it as a number between 1 and 10, multiplied by a power of 10	Examples: $408 = 4{,}08 \times 10^2$ $1\,300 = 1{,}3 \times 10^3$	$4{,}08 \times 100 = 408$ $1{,}3 \times 1\,000 = 1\,300$

This picture shows the distance between earth and the nearest star; Alpha Centauri



40 350 000 000 000 000 km
In scientific notation this is : $4{,}035 \times 10^{13}$

Time needed
45 minutes

DICTIONARY:

express – represent

Trainer's Note:
Go over the answers together and make time for learners to re-do any of their calculations if necessary.

Activity 4:
Using scientific notation for large numbers

Work in pairs

1. Write each of these numbers in scientific notation:
   a. The Sahara Desert is 8 400 000km^2 in area.
   b. The Australian Desert is 1 550 000km^2 in area.
   c. The Kalahari Desert is 520 000km^2 in area.
   d. The Pacific Ocean is 165 384 000km^2 in area
   e. The Indian Ocean is 73 482 000km^2 in area with a depth of 3 809m

2. The sun's temperature at its centre is $1,4 \times 10^7$ degrees Celsius. Write this number in full.
3. Which star is further away: one that is 403×10^8km away or one that is 403×10^5km away? How many times further?

3. Decimal numbers as powers.

So far you have looked at how to write whole numbers using powers. But you can also express decimal values as powers. In this case you use negative exponents to do this. Look at the first few examples in the table in Activity 5.

Trainer's Note:
To follow this next activity learners must be confident in being able to read and interpret decimal values. Refer to Unit 8 for practise examples you think might be useful to do before proceeding with this activity.

Activity 5:
Writing decimal values as power

Work in pairs

1. Work with a partner to complete the following table:

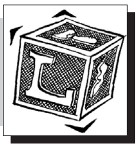
	Power of 10	Simplified answer
a	10^2	100
b	10^1	
c	10^0	
d	10^{-1}	
e	10^{-2}	
f	10^{-3}	
g	10^{-4}	

2. Write each of these fractions as a:
   (i) a decimal (ii) a power of ten
   a) $\frac{1}{10}$ b) $\frac{1}{100}$ c) $\frac{1}{1000}$ d) $\frac{1}{1\ 000\ 000}$ e) $\frac{1}{10\ 000\ 000}$ f) $\frac{1}{100\ 000\ 000}$

> ***Trainer's Note:***
> *Review answers together and note what difficulties your learners might have had in doing these exercises correctly. You may decide to go straight on to the unit on decimals if, for example, the major problem arose because of their not being able to work with decimals.*

3. Here are some examples of where you would use negative powers to describe measurements in our physical world.
   Convert each number to a (a) common fraction b) a decimal fraction

   a) Radio waves are 10^{-4} metres long
   b) The light rays of visible light are 10^{-6} metres long
   c) The size of an atom is 1^{-10} metres long

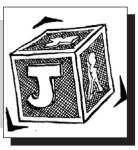


What have you learned?

Mathematicians have always tried to find short and concise ways of describing numbers and number relationships. Power notation is an example of an efficient way to write very large or very small numbers. Power notation makes it easy to interpret at a glance the value of these numbers. Once you understand the system of place value and know the value of a number like one million, you will quickly recognise the number written as a power 10^6. You can use this short notation yourself. You only need to write 3 digits to express the value of the number which otherwise you would use 7 digits to write in full as 1 000 000.

You can also use scientific notation to represent a very large measurement. A number like 8 897 000 000 000 000 km can be written as 8,897 × 1012. As some calculators cannot deal with numbers of more than 6 digits It is helpful to re-write these numbers in power notation to calculate their value.

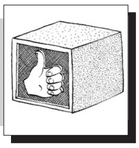
The more you advance mathematically, the more you will use power notation to write numbers and spatial relationships and to do calculations. In this manual you will use power notation again in the next unit on square and cubed numbers, in later modules on shape and space, and in the patterns module.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about powers and scientific notation?
b. How do you think you will be able to improve your skills of working with powers?
c. Write down one or two questions that you still have about powers or scientific notation.
d. How do you think you will be able to improve your scientific notation?
e. How will you use what you learned about powers and scientific notation in your everyday life and work?

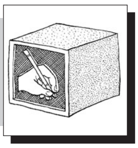


Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Use power notation to write large and small numbers in short form.				
2. Explain what the terms power, base and exponent mean				
3. Interpret a range of different numbers expressed in powers, not only those written to the power of 10.				
4. Identify and interpret examples in the real world where power notation is used as a way of expressing larger or smaller numbers				
5. Express large measurements using scientific notation				



Assignment 2: Large and small numbers in the world around you

Do some of your own research by using the library, or the internet to find at least ten examples that give measurements of aspects of our physical world in very large or very small numbers For example the size or distance between planets, the area of space covered by land or sea, the depth of different oceans or the size of a virus. Write each measurement down in long form and then using scientific notation to express the same measurement in short form.

UNIT FOUR

Square and Cube numbers

In this unit you will address the following:

Unit Standard 7447

SO1:

Express and interpret a range of contexts using mathematical symbols and find applications for numerical models. (everyday problems, numerical contexts; numerical models include equations, expressions and terms).

SO2:

Solve a range of everyday problems using estimation and calculations. (Rounding off and truncating, with or without calculator, combination, separation, comparison, equalization, sharing and grouping).

SO4:

Perform operations on simple and complex numerical expressions. (four basic operations in all combinations; expressions involving exponents that can be calculated without a calculator; operations to be performed with and without a calculator).

SO6:

Critically analyze the development of the base ten number system. (Place value, role of 0 in our number system, patterned nature of whole numbers, history and contestations).

SO7:

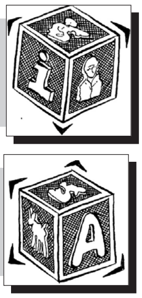
Analyze the relationship between rational and whole numbers.

SO8:

Analyze the relationship between rational numbers and integers.

To do this you will:

- recognize and explain what a square number is;
- identify, explain and predict patterns in a sequence of square numbers;
- explain links between square numbers and square shapes;
- explain the meaning of square root and the symbol used; do both mental calculations and use a calculator to estimate and find the square root of numbers that are both perfect squares and that are not perfect squares;
- interpret the notation used to describe the area of a square; do related area calculations;
- recognise and explain what a cubic number is;
- identify, explain and predict patterns in a sequence of cubic numbers;
- explain links between cubic numbers and cubed shapes;
- explain the meaning of cube root and the symbol used; do both mental calculations and use a calculator to find the cube root of unknown numbers;
- interpret the notation used to describe the volume of a cube; do related volume calculations.



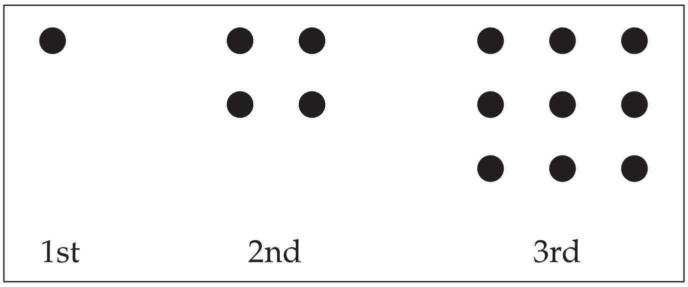
Time needed
70 minutes

1. Square Numbers

Activity 1: Investigating square numbers

Work alone

Some numbers can be shown as a square pattern of dots.



1. Copy these first three square numbers and then draw the next 4 square numbers with dots.
2. Copy and complete this table

Pattern in number	1	2	3	4	5	6
Total number of dots	1	4	9			

3. Can you spot a pattern in the numbers in the second row?
4. If the 26th square number in the pattern has 676 dots, use the pattern you noticed above to say how many dots there will be in the i) 25th pattern ii) 27th pattern.
5. Draw dots to see if these are square numbers or not a) 28 b) 34 e) 49
6. Experiment with the square root sign on your calculator to find out if a particular number you think of is a square number or not (Your answer must be a whole number).

Share your findings with a partner.

7. How many dots will there be in a square that has 18 in one row?
8. Look at the sequence of square numbers in the table below to see if you can find a secondary pattern in the size of the square numbers from the 1st one to the 5th one.

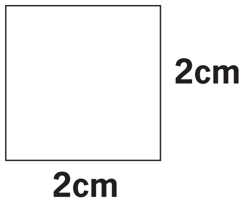
1st square number	2nd square number	3rd square number	4th square number	5th square number
1	4	9	16	25

9. Test to see if your pattern works by naming the next 5 square numbers in the sequence.
10. The square of 12 is 144. The square of 13 is 169. Use the pattern you found above to predict what the next 4 square numbers will be. Now test to see if you were correct by using your calculator to find the next 10 squares after 13.

What have you learned?

To square a number you multiply the number by itself for example the square of 3 is $3 \times 3 = 9$. Another way to write this is 3^2. This means 3, multiplied by itself.

Square numbers can be linked to square shapes. A square is a closed shape with straight sides where all sides are equal. You can find its area by multiplying one side by another. Your answers are expressed in square units. These could be anything from mm^2 to km^2 or even square hectares.



For example in this square the sides are 2cm. The area is 2×2 which is $4cm^2$. The square of 2 is 4.

Activity 2:
Finding the square root

Time needed
60 minutes

Work alone

1. Find the area of these squares that have the following lengths of sides:

   (i) 7m (ii) 9cm (iii) 11cm (iv) 100cm

2. Find the length of sides in squares that have the following areas:

   (i) $9cm^2$ (ii) $125cm^2$ (iii) $64m^2$ (iv) $144m^2$ (v) $10\,000km^2$

3. Check all your answers with a partner and correct any mistakes you may have made.

2. The Square Root

The opposite of squaring a number is called finding the square root. The symbol for square root is $\sqrt{}$. So the $\sqrt{25}$ is 5 because $5 \times 5 = 25$.

Trainer's Note:
Let learners spend time investigating how to use the square root using their calculators.

Activity 3:
Finding the square

Time needed
55 minutes

Work alone

1. Find the following
   a) $\sqrt{49}$ b) $\sqrt{9} + \sqrt{16}$ c) $\sqrt{144} - \surd\sqrt{64}$ d) $\frac{\sqrt{16}}{\sqrt{4}}$ e) $\frac{\sqrt{25}}{\sqrt{81}}$

2. First estimate then use your calculator to find the square root of these numbers. Round your answer to the nearest tenth.

   (i) $\sqrt{46}$ (ii) $\sqrt{20}$ (iii) $\sqrt{38}$ (iv) $\sqrt{110}$ (v) $\sqrt{139}$ (vi) $\sqrt{42{,}5}$

3. Discuss and compare solutions with a partner

What have you learned?

You can use the square root sign $\surd$ on your calculator to find the square root of any number.
The square roots of all the numbers above are whole numbers. You know that $7 \times 7 = 49$ so you can work out $\sqrt{49}$ as 7. Even for e) the answer is a whole number divided by a whole number, $\frac{5}{9}$

The square root will be a whole number if it is the square root of a perfect square, 12×12 or 8×8 etc.

But you can find the square root of other numbers too. The answer will fall somewhere between the square root of perfect squares. For example, you know that 12 is not a square number but it lies in between the two square numbers 9 and 16. So you could estimate that the square root of 12 will be smaller than 4 (the square root of 16), but bigger than 3 (the square root of 9).

Activity 4:
Cubic Numbers

Time needed
40 minutes

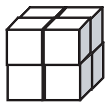
Work in pairs

1. Think about what you know about square numbers.
2. Discuss with a partner what you think a cubic number is. What shape do you think a cubic number is linked to?

3. Cubic Numbers

The cube of any number is the number multiplied by itself and then multiplied again. So, the cube of 2 is $2 \times 2 \times 2 = 8$.

Remember that square numbers can be linked to the shape of a square with all sides the same length. Cubic numbers link to the shape of a cube where all three dimensions, length, breadth and height, are the same. So 4 multiplied by itself, three times, is $4 \times 4 \times 4 = 64$. You call 64 a cubic number. The short form of this is to write $4^3 = 64$. You say "four cubed is 64".

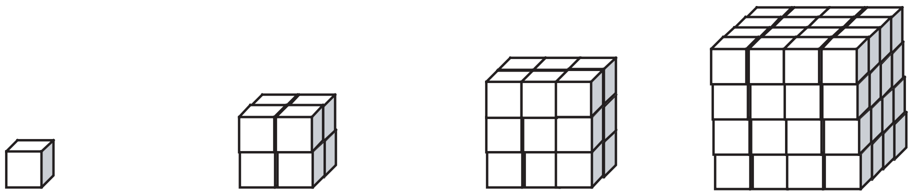


Activity 5: Cubic numbers

Time needed 40 minutes

Work alone

1. Study these diagrams of four cubes.



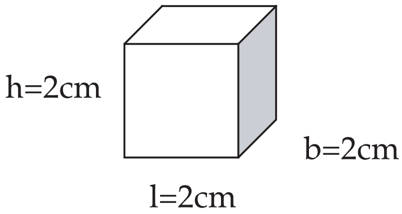
1×1 cube	2×2 cube	3×3 cube	4 x 4 cube
Cube 1	Cube 2	Cube 3	Cube 4

2. Now copy and complete the table below:

Length of on side of the cube	1	2	3	4	5
Number of small cubes					

3. Calculate the following
   a) 6^3 b) 3^3 c) 1^3 d) 7^3 e) 9^3 f) 12^3

4. Cubic numbers and volume



Imagine this is a block of cheese. The amount of cheese is the volume of cheese. The volume of cheese is $2 \times 2 \times 2$ cm which is 8cm^3 of cheese. The unit of volume in this case is a cubic centimetre which you write as cm^3.

Time needed
30 minutes

Activity 6:
Cubic numbers and volume

Work alone

1. Calculate the volume of the following cubes:
   a. A water tank whose width is 2m
   b. A box of sweets whose length is 10cm
   c. A box of chalk whose height is 8cm

5. Cube roots

When you are given the volume of a cube you can find the length of one of the sides by finding the cube root of the volume.

The symbol for cubed root is $\sqrt[3]{\ }$. You can see the difference between this and the symbol $\sqrt{\ }$ for square roots.

Look at these two examples:

$\sqrt[3]{8}$ is 2 because $2 \times 2 \times 2 = 8$

$\sqrt[3]{27}$ is 3 because $3 \times 3 \times 3 = 27$

Time needed
50 minutes

Activity 7:
Find the cube root

Work alone

1. Copy and complete this table to find the first ten cubic numbers and their cube roots.

Cubic Numbers	1	8	27							
Cube roots	1	2	3	4	5	6	7	8	9	10

2. Calculate:

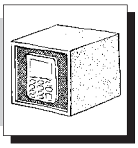
   a) $\sqrt[3]{270}$ b) $\sqrt[3]{1000}$ c) $\sqrt{36} \times 9$

   d) $\sqrt[3]{125}$ e) $\sqrt{400} + 4^2$ f) $(2 \times 3) + 2 + 3^2$

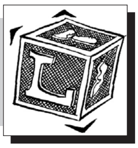
3. a.Find the length of the sides of cubes with these volumes in cubic millimetres.

Volume	1 000mm^3	270mm^3	2160mm^3	64000mm^3
Length of each side				

4. Change each your answer in a. to cm^3 in each case.



5. Some calculators have cube root signs. Find one that does and use it to find cubes and cube roots and cubes of even larger numbers than the example used here.

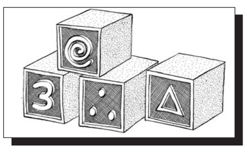


What have you learned?

A cubic number is a number multiplied by itself twice; e.g. $2 \times 2 \times 2$. you can use power notation to write a cubic number e.g. 2^3 where 2 is the base and ^{3} is the exponent

- Cubic numbers link directly to cubed shapes. A cube is a 3 dimensional shape where all 3 dimensions; length, breadth and height are equal. To find the volume of a cube you can multiply any dimension by itself twice; i.e. s^3, where s = the length of side.
- So if you know the length of one side you can find the volume. Similarly if you know the volume of any given cube, you can find the length of its sides. In number terms if you know the cubic number you can find the cube root.
- We can use the cubed root sign $\sqrt[3]{}$ on your calculator to find the cube root of any number.
- Only perfect cubes will give a whole number as the answers.
- It is useful to memorise the first 10 or so cubed numbers and cubed roots.

Trainer's Note:
Review answers to the section together. Assess whether your learners need more support. If they do design additional practise activities based on the activities they will have worked through.

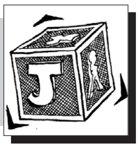


Linking your learning with your ECD work

Building squares

- Talk with children about how some shapes are the same and how they are different. Ask questions that help the children to discover that a square is different in that all its sides are the same length. Let the children touch and hold the squares.
- Children can build the outline of a square, with sides the same length, using buttons or round counters. Add counters to the length of one side and ask the children to describe what has happened to the shape and what you need

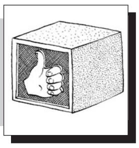
to do to the other sides to build a bigger square. The children should be able to explain that you need to add the same number of counters to the other three sides to make a bigger square. They can make bigger and bigger squares using buttons or counters or similar construction materials.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about square and cube numbers?
b. How do you think you will be able to improve your understanding of square and cube numbers?
c. Write down one or two questions that you still have about square and cube numbers.
d. How will you use what you learned about square and cube numbers in your everyday life and work?

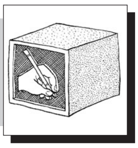


Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

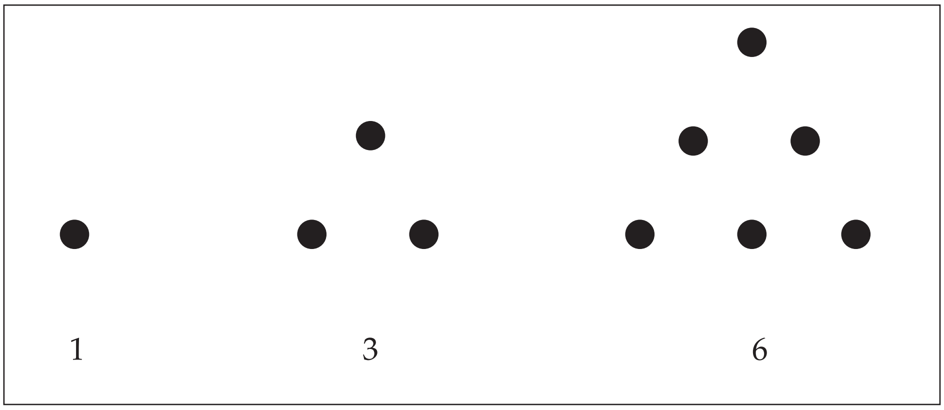
I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Recognise and explain what a square number is				
2. Identify, explain and predict patterns in a sequence of square numbers				
3. Explain links between square numbers and square shapes				
4. Explain the meaning of square root, the symbol used; do both mental calculations and use a calculator to estimate and find the square root of numbers that are both perfect squares and that are not perfect squares.				
5. Interpret the notation used to describe the area of a square; do related area calculations				
6. Recognise and explain what a cubic number is				
7. Identify, explain and predict patterns in a sequence of cubic numbers				
8. Explain links between cubic numbers and cubed shapes				
9. Explain the meaning of cube root and the symbol used; do both mental calculations and use a calculator to find the cube root of unknown numbers				
10. Interpret the notation used to describe the volume of a cube; do related volume calculations				



Assignment 3:

You have investigated square and cube numbers and patterns. Numbers can form other patterns too. A triangular number can make the shape of a triangle.



Extend the pattern to find all the triangular numbers up to 78.
Find a pattern between the difference in the number of dots each time.
Use what you find out to predict the next 10 triangular numbers after 78.
Share your findings with a partner.

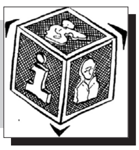
UNIT FIVE

Negative Values and Measures

In this unit you will address the following:

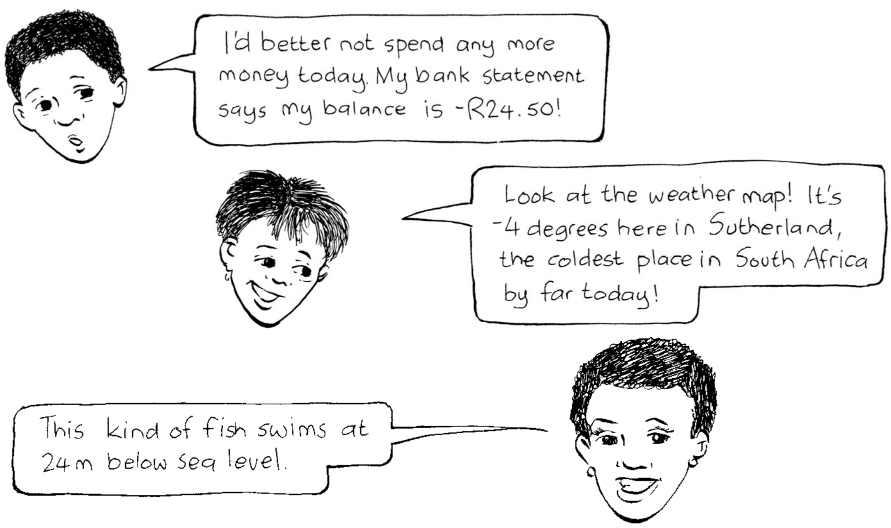
Unit Standard 7447
SO8:
Analyze the relationship between rational numbers and integers.

To do this you will
- work with zero as the point of origin (starting point) for both negative and positive numbers;
- explore negative numbers that measure values less than zero and positive numbers measure values more than zero;
- work with some practical situations where we use negative numbers in daily life;
- explore how to add on from both positive and negative numbers.



1. Thinking about zero

Think of 0 as your starting number. This can also be called the point of origin. Read each of the following things that the people are saying. Discuss how each of these relates to zero.



You can you see that in all the examples something is measured below or less than zero. Long ago, mathematicians found an easy way to describe numbers less than 0. They had the idea of negative numbers. Negative numbers are written with a – sign in front of them. Negative whole numbers, such as –4, –198 and so on, also belong to a set of numbers we call integers.

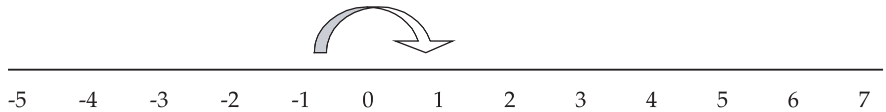
Trainer's Note:
Invite learners to think of more examples of their own where we use negative numbers in everyday situations and to think about the benefits of this system.

Time needed
70 minutes

Activity 1:
Reading positive and negative numbers on a number line

Work in pairs

Number lines are a helpful way to help us to think about number relationships and do calculations with negative and positive numbers.



For example, from the number line above, can you see how to work out the difference between -1 and 1 to get an answer of 2?

1. Use what you found out on the number line to calculate the difference between all the other opposite numbers shown on the number line. (i.e -2 and 2, -3 and 3, and so on).
2. Write down all the whole numbers on the number line above that are:
   a. Larger than -4 and smaller than 3.
   b. Smaller than 7 but smaller than -4

3. Draw your own number line that starts as -12 and ends at +12. Use the number line to count on from the given number to find the following:

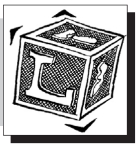
a) -12 + 4 =	b) -7 + 6 =	c) 2 + 5 =	d) 0 + 5 =	e) -7 + 12 =
f) -8 + 12 =	g) -10 + 10 =	h) -9 + 6 =	i) -2 + 7	j) -6 + 10 =
k) -6 + 6 =	l) -3 + 5 =	m) -9 + 2 =	n) -7 + 4 =	o) -3 + 3 =

4. Order these numbers from the smallest to the biggest.
   a. 2; -4; 0; -7; 4; -11; -3; 14; -8; 7
   b. 44; -27; 38; 4; -5; 0; 17; -13; -36

5. Which number is?
   a. 31 more than -17
   b. 400 more than -50
   c. 16 less than -120

6. Start at -28. Add on in 4s until you reach 36. Write the numbers you get each time.
7. Start at 15. Subtract in 5s until you get to -55. Write the numbers you get each time.

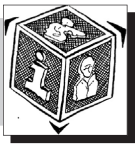
Trainer's Note:
Review answers together. Let learners make any corrections necessary. Give more practice in similar activities if you feel this may be necessary. Also go through the steps of checking these calculations using their calculators making sure that they enter the correct operational sign in the correct order.



What have you learned?

By doing these activities you had practice counting forwards and backwards from negative to positive numbers and visa versa. So for example -2 + 2 are 4; - 4 plus 4 gives you 8. You found out how a number line is useful to think about and count negative and positive values with 0 as the starting point. Maybe now you can do more calculations without drawing a number line.

To find the number that was 31 more than -17 (5a), you first count 17 up to 0 and then add 31 to get an answer of 48. If you are still unsure, work with a partner and make up questions to ask each other. You can use a calculator to check your answers or draw the numbers on a number line.



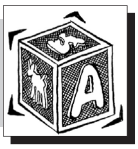
2. Negative numbers in money

Think about money you have borrowed and the amount you owe. Maybe you have an agreement to pay back a certain amount over a period of time. As you pay it back you owe less money.

For example:

Thuli lent Lebo R75. Lebo agreed to pay her back an amount of R7.50 every week. This means that once Lebo borrowed the money, she started with a negative balance of –75. When Lebo paid back R45.00 after 6 weeks, her balance was at –R30.00. Finally after 10 weeks it was zero.

If you have an overdraft at the bank it means you can borrow money from the bank and spend money even if you have a zero balance. As you spend money on the overdraft your account has a negative balance. As money is paid in to the account the negative balance gets smaller until the balance is zero, or even a positive balance.



Time needed
30 minutes

Activity 2:
What's the balance?

Work alone

Bella's Bakery has on overdraft of R20 000. This is what their bank statement looks like in Mid June:

+ 21 683	24/5
- 4 087	6/6
- 11 921	8/6
- 8 076	15/6
	16/6

a. Calculate Bella's Bakery's expenses to find what the balance was on the 16th June.
b. How much of the overdraft amount is left?
c. The bakery made a payment of R11 250.45 at the end of June. What figure appeared on their bank statement after this payment?
d. The next amount on their account showed an income of R32 567.05. What was the balance after this transaction? Remember to think about the overdraft amount.
e. Discuss and compare answers with a partner

What have you learned?

Balance and overdrafts are good examples to show where negative numbers are used in daily life. If your bank balance shows a negative amount it means you have owe the bank that amount of money.

3. Measuring temperatures

Temperatures can be above or below 0. Temperatures are measured in degrees. We write it like this °. When you calculate differences between minus zero and above zero temperatures you are also working with positive and negative numbers. There are two systems for measuring temperature: Celsius and Fahrenheit. In South Africa we use the Celsius scale. In this scale the freezing point of water is 0 °C and the boiling point 100 °C.

Activity 3:
Measuring Temperature

Time needed
70 minutes

Work alone

Answer the following questions about temperatures:

1. In each case, which temperature is hotter and by how many degrees??
   a) -4°C and 9°C b) -5, 5°C and 0°C c) 13, 2°C or -2, 8°C

2. In each case, which temperature is colder and by how many degrees?
   a) +1°C or +7 °C b) +3 °C or -5 °C c) 1 °C or -6 °C

3. If a thermometer starts at -5°C, and rises by 6 degrees, the thermometer will show 1°C. What will the thermometer show if the temperature:
   a) Starts at +2°C and rises by 5 degrees b) Starts at -2°C and rises by 7 degrees

4. By how many degrees does the temperature rise when it goes from?
   a) +7°C to +11°C b) -3°C to +2 °C
   c) -6°C to 10°C d) -5°C to -1°C

5. By how many degrees does the temperature fall when it goes from?
   a) +5°C to +2°C b) 0°C to -6 °C c) -4°C to -5°C

What have you learned?

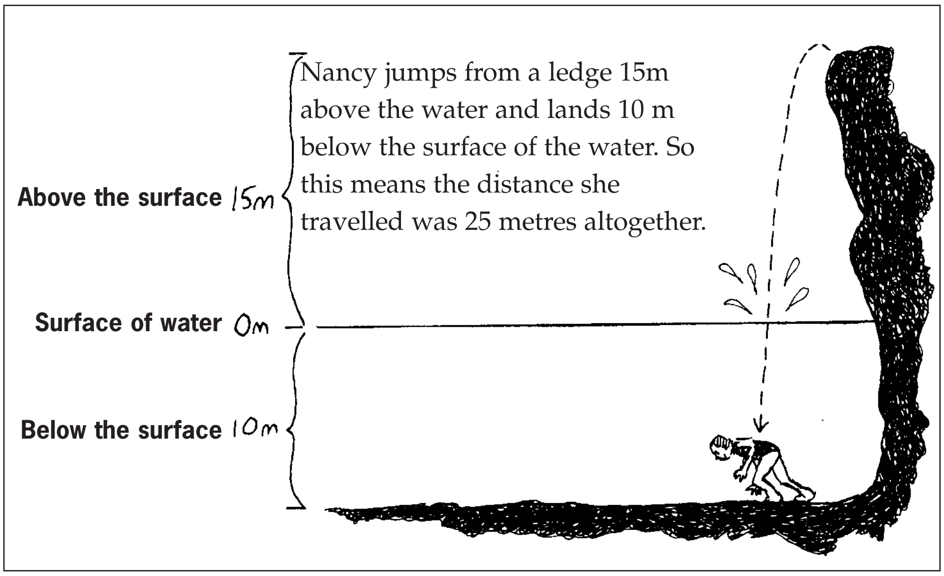
You did these calculations in the same way that you worked with positive and negative numbers in Activity 1 and 2. So to find out that 9 ℃ is hotter than -4℃ by 13℃ you first counted up 4 degrees to 0 ℃, and then added 9 ℃ to get to 13 ℃ as your answer. In the same way the temperature falls 1° when it goes from -4 ℃ to -5 ℃ because you count down 1 from -4 to -5.

Activity 4:
Above and below surface level.

Time needed
50 minutes

Work alone

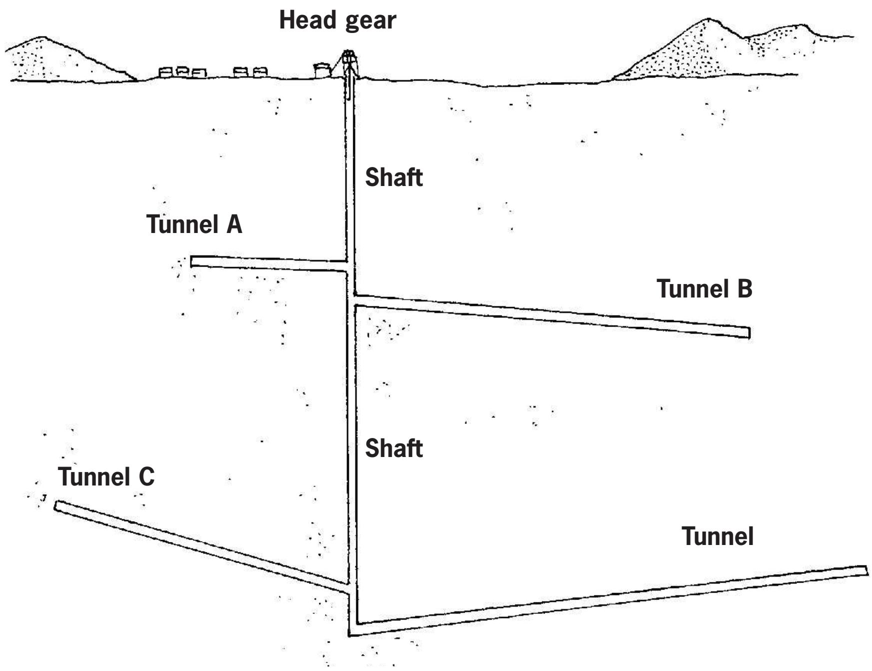
1. Look at the sketch below and think about the distance Nancy travels:



2. Draw a vertical number line to help you find the distance from:
   a. 35m above the surface of the water level to 13,5m below.
   b. 12m above the surface to 17,5m below the surface.
   c. 3,5m below the surface to 12,2m above.

Look at the next example of mining. In this picture of a mineshaft, every cm in the picture represents 0,25km or $\frac{1}{4}$ km or 250m in real life.

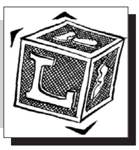
Scale: 1cm: 0,25km



3. Use a ruler and the given scale to find the depth of each tunnel.
4. How many more kilometres underground is
   a. Tunnel D than Tunnel C
   b. Tunnel C than Tunnel A

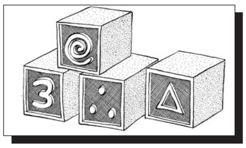
5. The headgear of the mind stands 55m above the ground. A rescue helicopter hovers just above the headgear. A miner is stranded at Tunnel D.
   a. What is the distance between where the helicopter is and Tunnel D?
   b. The miner is rescued and flown to safety. The helicopter flies 125m above the ground. The helicopter then travels to a hospital 4km away. How many km and m has the miner traveled from the Tunnel D to the hospital?

6. Share your answers with a partner. Make corrections if you need to.



What have you learned?

We can use negative numbers and positive numbers to calculate from a point above the surface of land or water to below the surface. When you talk about a length or distance of 10m below the surface you do not write -10m. But in fact those measurements are negative values. So if Nancy dived from a height of 15 m above the surface of the water, to 10m below the surface, you count up from -10 to 0 which is 10m, and then add 15m to get to 25m. A shorter way is to add 10 to 15 to get to your answer.



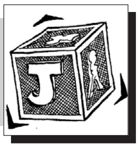
Linking your learning with your ECD work

Young children use positive and negative numbers, usually without realizing. For example, in borrowing and lending games:

- Jane and Agnes are playing with blocks and Jane borrows five long blocks from Agnes and promises to give them back. She gives 2 back and then "owes" Agnes 3.

- Or in a shopping game where they use play money to buy goods and owe money if they do not have enough to pay.

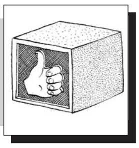
Encourage the children's own language to talk about and explain these ideas when they are playing.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about zero?
b. What did you learn from this unit about negative numbers?
c. Write down one or two questions that you still have about positive and negative numbers.
d. How do you think you will be able to improve your understanding of negative numbers?
e. How will you use what you learned about zero, positive and negative numbers in your everyday life and work?



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. That zero is the point of origin for both negative and positive numbers				
2. That negative numbers measure values less than zero and positive numbers measure amounts or values more than zero				
3. About some practical situations where we use negative numbers in daily life				
4. How to do calculations involving both positive and negative numbers.				

UNIT SIX

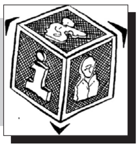
Fractions

Trainer's Note:
People tend to use misleading language when talking about fractions. While working through the Unit, try to encourage the use of correct and precise fraction language. For example, to refer to one quarter as "one over 4", should be discouraged as it has no real meaning. What would be more precise would be one "out of four", because when we name fractions we are describing one out of four parts.

In this unit you will address the following:

Unit Standard 7447
SO7:
Analyze the relationship between rational and whole numbers.

To do this you will:
- describe different kinds of fractions;
- find equivalent fractions by using your knowledge of multiples and factors;
- reduce fractions to their simplest form; convert improper fractions to mixed fractions;
- so calculations with fractions including addition subtraction, multiplication and division;
- Solve fraction problems in context, choosing the correct operation to find the answer.



1. Naming parts of wholes

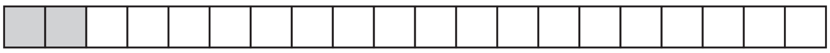
When something is divided into 5 equal parts, each part is called one fifth of the whole.
When something is divided into 6 equal parts, each part is called one sixth of the whole.

one sixth	one sixth	one sixth	one sixth	one sixth	one sixth

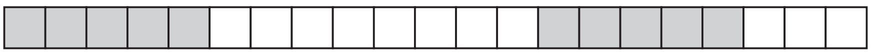
Time needed
45 minutes

Activity 1:
Naming parts of wholes

Work alone
1. Which is more, one fifth of a loaf of bread, or one sixth of a loaf of bread? Why?
2. Which is less $\frac{1}{35}$ or $\frac{1}{42}$?
3. When something is divided into 20 equal parts, each part is called a twentieth of the whole:
   a. What fraction of the diagram below is shaded?
   b. What fraction is unshaded?

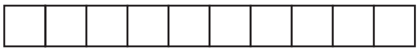


4. What fraction name do we give to each block in the diagram below?
5. What fraction is shaded?



Trainer's Note:
Allow time for your learners to discuss and compare answer.

6. Make a diagram to demonstrate what is meant by four fifteenths.
7. Shade 4 blocks in the diagram below.



   a. What fraction have you shaded?
   b. Gerard says that he shaded $\frac{4}{5}$. Dudizile says she shaded $\frac{8}{10}$. Are they both right? Explain your thinking to a partner.

What have you learned

The fraction name we give to one of the shaded blocks depends on the number of equal blocks we have divided the strip into i.e $\frac{1}{20}$ or $\frac{1}{10}$. When writing a common fraction, the denominator or bottom number tells us how many parts the whole has been broken up into. The numerator or top number tells us the number of parts we are naming.
In the discussion that Gerard and Duduzile had, Duduzile was right: $\frac{8}{10}$ is the same as $\frac{4}{5}$. This leads us to the next investigation - understanding equivalent fractions.

2. Equivalent Fractions

Activity 2:
Equivalent Fractions

Time needed
50 minutes

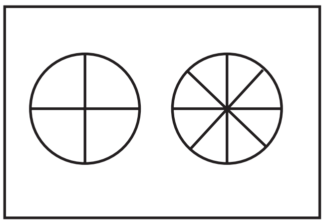
Work alone

Peter and Buthi were having an argument about who got more pie.

Peter: It's not fair Buthi! I got $\frac{1}{4}$ of this pie and you got $\frac{2}{8}$.

Buthi: But can't you see Peter? $\frac{1}{4}$ is the same as $\frac{2}{8}$.

Buthi did a drawing for Peter to show him she was right.



1. Do you agree with Buthi? Use your ruler and pencil to draw two rectangular blocks like the ones on the next page and divide them into fractions to prove that $\frac{1}{4} = \frac{2}{8}$

2. Write down at least three other fractions that are equal to $\frac{1}{4}$.
3. Discuss and compare answers with a partner.

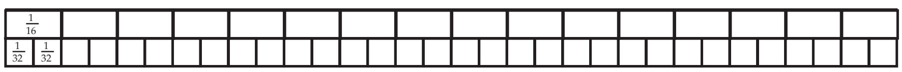
4. Dudu did a calculation to prove that $\frac{1}{4} = \frac{2}{8}$ as follows:

   $$\frac{2 \div 2}{8 \div 2} = \frac{1}{4}$$

   Annah wrote it like this:
   $1 \times 2 = 2$
   $4 \times 2 = 8$

   a. Explain why both ways are correct?
   b. Use either method above, or any other method you know, to find at least (not less than) four different fractions that have the same value as these ones:

   a) $\frac{1}{5}$ b) $\frac{2}{3}$ c) $\frac{3}{4}$ d) $\frac{3}{8}$

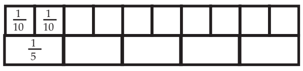


5. You can see the picture above that $\frac{1}{16}$ is the same as $\frac{2}{32}$. Use the fraction wall to find equivalent fractions for these fractions:

   a) $\frac{4}{32}$ b) $\frac{10}{16}$ c) $\frac{24}{32}$ d) $\frac{9}{16}$ e) $\frac{20}{32}$ f) $\frac{14}{16}$

What have you learned?

Equivalent fractions have the same value but are expressed differently, with different sized numerators and denominators – but in the same relationship to each other. So in Duduzile's example in question, where she said that $\frac{4}{5}$ was the same as $\frac{8}{10}$ she might have explained this by multiplying both the numerator and denominator by 2, or by doing a drawing where she could show this by dividing the same sized rectangle into 10^{ths} and then 5^{ths}.



You probably found the task above easy because you had a picture to guide you. But what do you do when you don't have a picture to look at? What method can you use that makes sense to find the equivalent fractions for any fraction?

From the picture above for example you noticed that $\frac{2}{16}$ was the same as $\frac{2}{16}$

So we could write: $2 \times 2 = 4$

We multiplied both the numerator and the denominator by 2.

In this example $\frac{4}{9} = \frac{12}{27}$

We multiplied both the numerator and the denominator by 3 to find this equivalent fraction.

Sometimes you may be given half the answer in an equivalent fraction calculation. Here is one easy way to find the missing value:

$\frac{6}{11} = \frac{?}{77}$

$\frac{6}{11} = \frac{42}{77}$

Multiply the numerator by the same number you multiply the denominator by, which in this case is 7.

Activity 3:
More Equivalent Fractions

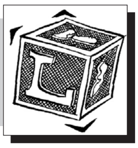
Time needed
45 minutes

Work alone

1. Use what you found out above to find the missing values in these equations:

   i) $\frac{6}{23} = \frac{48}{?}$ ii) $\frac{1}{2} = \frac{}{78}$ iii $\frac{16}{100} = \frac{?}{25}$ iv) $\frac{48}{36} = \frac{?}{6}$ v) $\frac{4}{3} = \frac{12}{?}$ vi) $\frac{1}{4} = \frac{9}{?}$

2. Discuss the difference between the fractions in example i) and iv) above.
3. Do you know the correct mathematical names for these two kinds of fractions?



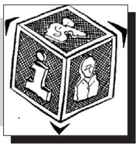
What have you learned?

You have now learned how to make two fractions equivalent by making the denominators or the numerators the same. You realized by doing these activities that it helps to have a good working knowledge of your multiplication and division facts. Once you can find equivalent fractions you can compare different fractions, find out which one is more or less, as well as add and subtract fractions with different denominators. You will get practice doing this in the next set of activities.

If you need more practice learning your basic multiplication and division facts, spend time doing this. There are quick ways to do this without having to memorise each fact. For example by using strategies of doubling and halving: If you know what 3×4 is, for example, you can double your answer to find 6×4 and double again to get to 12×4.

Refer back to Unit One to give yourself more practice in doubling and halving.

In these examples you also came across examples where the numerator was smaller than the denominator. We call these fractions proper fractions. You also found cases where the numerator was larger than the denominator; we call these improper fractions and can change them into mixed fractions so $\frac{48}{36}$ is the same as $1\frac{12}{36}$ or $1\frac{1}{3}$.



3. Comparing fractions

Trainer's Note:
It is important for learners to know their basic multiplication and division facts to do operations with fractions and to find equivalent fractions. You need to assess whether these skills are in place, and if not to consider what additional practice your learners may need that would help them to work through these sections.

If you have a good knowledge of your multiplication and division facts, it's easy to compare fractions with the same denominators. For example we know that $\frac{1}{8}$ is less than $\frac{2}{8}$ and $\frac{3}{8}$, but is it more than $\frac{5}{6}$?

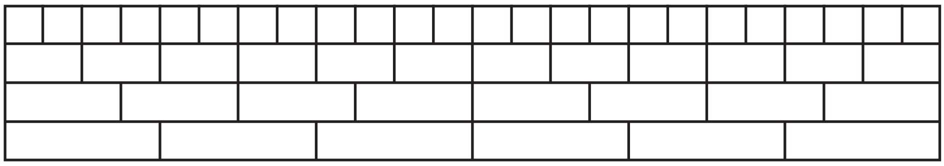
To compare fractions that have different denominators, you can change them into equivalent fractions with the same denominator. Here's a way to do this:

- List the multiples of 8: 16; 24; 32; 48; 56; 56; 64
- List the multiples of 6: 12; 18; 24; 30; 36; 42; 48
- Use these lists to find the smallest number that both 3 and 8 divide into – in this case it's 24. This is called the lowest common multiple (LCM).
- Change each fraction to have a denominator of 24 to find equivalent fractions:

$$\frac{3}{8} \underset{\times\mathbf{3}}{\overset{\times\mathbf{3}}{=}} \frac{9}{24} \qquad \frac{5}{6} \underset{\times\mathbf{4}}{\overset{\times\mathbf{4}}{=}} \frac{20}{24}$$

- So now you can tell that $\frac{5}{6} = \frac{20}{24}$, and is more than $\frac{3}{8}$, which $= \frac{9}{24}$

You can check that this is correct by reading the values of the fractions on this fraction wall.



Time needed 80 minutes

Activity 4: Comparing fractions

Work alone

1. Use this method or any other you know of to find the bigger fraction in each case:
   a) $\frac{3}{7}$ or $\frac{2}{3}$ b) $\frac{4}{9}$ or $\frac{5}{7}$ c) $\frac{3}{8}$ or $\frac{1}{3}$ d) $\frac{5}{11}$ or $\frac{7}{10}$
   e) $\frac{2}{9}$ or $\frac{3}{11}$ f) $\frac{6}{15}$ or $\frac{7}{10}$ g) $\frac{5}{12}$ or $\frac{7}{9}$ h) $\frac{3}{4}$ or $\frac{5}{6}$
2. Compare and correct answers with a partner.
3. Buthi says his way to find the LCM is to multiply the denominators together. For example, $\frac{3}{7}$ or $\frac{2}{3}$ so 21 is the LCM. But is this always true?
4. Test by using Buthi's method to see if the answer you get is the LCM in all cases for these sets of fractions
   a) $\frac{3}{9}$; $\frac{4}{7}$ b) $\frac{2}{5}$; $\frac{7}{10}$ c) $\frac{2}{3}$; $\frac{7}{10}$
5. Discuss and compare findings in your groups.

4. Writing fractions in their simplest form

When you want to write fractions in their simplest form, you need to find the highest common factor (HCF). The HCF is the biggest number or factor that you can divide into both numbers, without leaving a remainder. Once again it helps to have a good working knowledge of your multiplication and division facts to do this quickly!

When Phindi works out how to reduce $\frac{4}{8}$ to its simplest form she thinks as follows:

- I'll Write out down the factors of 4 and 8
  2, 4 and 2, 4, 8
- Then I'll find the HCF of both numbers = 4
- Now I can divide both numbers in the fraction by 4 : $\frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$ so $\frac{4}{8} = \frac{1}{2}$

Can you explain Phindi's thinking? Share ideas with a partner.

Time needed
10 minutes

Trainer's Note:
Learners may be able to do calculations like these mentally, without needing to write down all the steps in this way.

Activity 5: Simplifying fractions

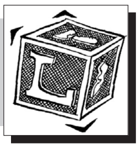
Work alone

1. Find the HCF in these sets of fractions and re-write them in their simplest form.

a) $\frac{9}{18}$	b) $\frac{36}{60}$	c) $\frac{18}{54}$	d) $\frac{16}{48}$
e) $\frac{11}{99}$	f) $\frac{60}{48}$	g) $\frac{32}{64}$	h) $\frac{27}{51}$
i) $\frac{120}{150}$	j) $\frac{200}{600}$	k) $\frac{360}{180}$	l) $\frac{96}{10}$

2. Find at least three equivalent improper fractions for the following:

a) $\frac{4}{7}$ b) $\frac{5}{6}$ c) $\frac{1}{2}$ d) $\frac{3}{4}$ e) $\frac{3}{11}$



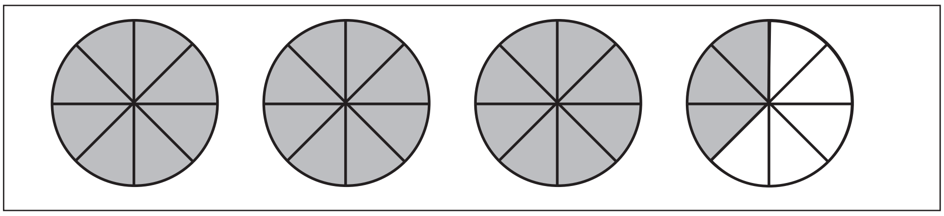
What have you learned?

To reduce a fraction to it simplest form you needed to use your knowledge of your multiplication and division facts to find the highest common factor HCF - the biggest number that you can divide into both the numerator and the denominator, without leaving a remainder.

Sometimes you don't recognise the HCF straight away and then take several steps to find the answer. For example in 1k) $\frac{360}{180}$ you may have thought of 10 first as being the HCF and reduced this fraction to $\frac{36}{18}$ and then seen that this could be reduced further, as 9 goes into both of these numbers to give you an answer of $\frac{4}{2}$. You could have reduced this even further on noticing that 2 can go into both 4 and 2, to give an answer of 2. Or you may have realized straight away that 180 was the HCF and divides into 360 twice to give you an answer of 2.

Can you see in the first method that each of the 3 numbers you divided the fraction by, multiplied by one another gets you to the HCF? $10 \times 9 \times 2 = 180$! The better knowledge you have of your multiplication and division facts, the fewer steps you will need to take.

5. Changing improper fractions to mixed numbers.



4 cakes are cut into 8 equal slices. Each slice is $\frac{1}{8}$ of a whole. The shaded parts show you how many slices were eaten. We can write this fraction in two ways. We could say $\frac{27}{8}$ have been eaten or $3\frac{3}{8}$ have been eaten.

$\frac{27}{8}$ is called an improper fraction. The numerator is bigger than the denominator $3\frac{3}{4}$ is called a mixed number. It shows both the whole cakes and the number of extra slices that were eaten.

Activity 6:
Working with mixed numbers and improper fractions

Time needed
90 minutes

Work alone

1. From the drawings on the previous page discuss:
   a. How you can change $\frac{27}{8}$ into $3\frac{3}{8}$.
   b. How you can change $3\frac{3}{8}$ into $\frac{27}{8}$.

2. Use what you found out in question 1 above to do the following:
   Change these improper fractions into mixed numbers

   a) $\frac{41}{4}$ b) $\frac{32}{3}$ c) $\frac{16}{5}$ d) $\frac{84}{9}$ e) $\frac{76}{7}$ f) $\frac{28}{6}$ g) $\frac{46}{3}$

   Change these mixed fractions into improper fractions.

   a) $1\frac{5}{6}$ b) $4\frac{3}{11}$ c) $6\frac{5}{8}$ d) $9\frac{3}{7}$
   e) $10\frac{4}{9}$ f) $11\frac{3}{10}$ g) $17\frac{2}{5}$

3. Which is the correct answer? $\frac{42}{12}$ is the same as:

   a) $3\frac{3}{12}$ b) $4\frac{3}{12}$ c) $3\frac{1}{2}$ d) $3\frac{7}{2}$

4. Which improper fraction is the same as $6\frac{3}{9}$?

   a) $\frac{27}{9}$ b) $\frac{57}{6}$ c) $\frac{54}{6}$ d) $\frac{57}{9}$

5. Which mixed number below is not the same as $\frac{4}{16}$?

   a) $2\frac{12}{16}$ b) $2\frac{1}{2}$ c) $2\frac{6}{8}$ d) $2\frac{3}{4}$

6. Discuss and compare answers in your group. Make any corrections necessary.

What have you learned?

In these activities you practised changing improper fractions to mixed numbers, and the other way around. Again you found that to do this quickly you needed to know your multiplication and division facts. Remember multiplication and division are related. This means that if you know for example, that 7×8 is 56 you also know that 56 divided by 7 is 8 or 56 divided by 8 is 7

6. Adding and subtracting fractions

In an office park there are two buildings with offices to let. $\frac{1}{3}$ of the space in one building is used by a security company. $\frac{2}{5}$ of the other building is used by an NGO. The rest of the space is empty. Liziwe works for the owners of the buildings. She has to give a report on how much space is vacant. This is how she thought about the problem:

To add $\frac{1}{3} + \frac{2}{5}$, I must make them equivalent.
The lowest common multiple (LCM) that both 3 and 5 divide into is 15 so

$= \frac{1}{3} + \frac{2}{5}$
$= \frac{5 + 6}{15}$

To find the total space used up I write $\frac{5}{15} + \frac{6}{15} = \frac{11}{15}$.

So that's the same as $\frac{2}{1} - \frac{11}{15}$ (Common denominator is 15)

so that's $\frac{30 - 11}{15}$

So that means $\frac{19}{15}$ or $1\frac{4}{15}$ of the space that's empty.

Activity 7:
Adding and subtracting fractions

Time needed
60 minutes

Work in pairs

1. Discuss Liziwe's method above. Make sure you understand all the steps.
2. Now use this method, or any other that you know of, to add or subtract these fractions:

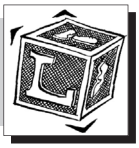
a) $\frac{1}{3} + \frac{3}{7}$ b) $2 - \frac{4}{7}$ c) $(\frac{6}{11} + \frac{4}{3}) - \frac{1}{4}$

d) $\frac{8}{9} + \frac{17}{18} + 3$ e) $4 - (\frac{13}{16} - \frac{5}{8})$ f) $(\frac{10}{30} + \frac{7}{15}) - \frac{1}{10}$

g) $2 - (\frac{4}{9} + \frac{6}{27}) + 1$ h) $12 - (\frac{24}{35} - \frac{6}{7})$ i) $34 - (\frac{23}{4} + 5\frac{1}{6})$

3. Two Bags of oranges are shared in this way: Pumzile gets $\frac{1}{8}$ of the share, Bheki gets $\frac{1}{4}$ of the share and Andile gets $\frac{2}{6}$ of the share.
   What fraction is their share altogether? What fraction is left over?

4. Work out the missing fraction to complete these number sentences:
   a) $\frac{5}{12} + \frac{10}{24} + \frac{?}{24} = 2$ b) $\frac{3}{16} + \frac{4}{8} + \frac{?}{16} = 3$



What have you learned?

Here are some steps to help you find the answer to an example like $4\frac{5}{8} - 1\frac{3}{4}$:

- Write the numbers as improper fractions first $= \frac{37}{8} - \frac{7}{4}$
- Find the LCM and do the subtraction

$$\frac{37}{8} - \frac{7}{4} \times \frac{2}{2}$$
$$= \frac{37}{8} - \frac{14}{8}$$
$$= \frac{23}{8}$$

- Write the answer as a mixed number $2\frac{7}{8}$
- Check that the answer is in its simplest form

Trainer's Note:
This is only one possible method. Your learners may know shorter methods that work better for them. Encourage them to share these with their colleagues.

Time needed
45 minutes

Activity 8:
Calculating with mixed numbers

Work alone

1. Calculate the following. Give your answer as a mixed number where you can.
   a) $2\frac{1}{4} + 4\frac{1}{6}$ b) $6\frac{3}{8} - 2\frac{2}{3}$ c) $4\frac{3}{7} - 2\frac{5}{8}$
   d) $9\frac{3}{10} + 6\frac{2}{3}$ e) $5\frac{6}{11} - 3\frac{2}{9}$ f) $14\frac{1}{2} - 8\frac{3}{8}$

2. Find a quick way to do these
   a) $4 + 4\frac{1}{4}$ b) $6 + 3\frac{1}{8}$ c) $9 + \frac{8}{15}$
   d) $10 + 2\frac{3}{4}$ e) $5 - \frac{1}{2}$ f) $10\frac{2}{3} - 7$

3. a) $(4\frac{1}{3} - 3\frac{2}{7}) + 6\frac{1}{4}$ b) $(12\frac{3}{25} - 3) + 1\frac{1}{2}$

4. a) $6\frac{1}{4} - ? = 5\frac{2}{5}$ b) $7\frac{1}{2} + ? = \frac{87}{16}$

5. Check and compare answers to 1-5 in a group. Do any corrections necessary.

7. Multiplication of fractions

Remember that multiplication means repeated addition. With whole numbers 4×3 means $3 + 3 + 3 + 3$. It is the same with $4 \times \frac{1}{3}$ which means $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$. You know that $4 \times \frac{1}{3}$ is the same as $\frac{4}{3}$, which simplified is $1\frac{1}{3}$.

Activity 9:
Repeated addition

Time needed
20 minutes

Work alone

1. Write these multiplication sentences as addition sentences. Simplify your answers where you need to.

   a) $3 \times \frac{3}{4}$ b) $2 \times \frac{1}{8}$ c) $4 \times \frac{3}{17}$ d) $8 \times \frac{1}{12}$

2. Write these addition number sentences as multiplication sentences. Find the answers.

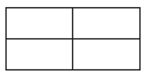
   a) $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ b) $\frac{2}{16} + \frac{2}{16} + \frac{2}{16} + \frac{1}{16}$

Activity 10:
Fraction table

Time needed
30 minutes

Work alone

1. Copy and complete this table.

Find the product of	Written in words	Shown in a picture	Shown as addition	Answer
$3 \times \frac{1}{4}$	Three quarters		$\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$	$\frac{3}{4}$
$4 \times \frac{1}{6}$				
$7 \times \frac{1}{2}$				
$4 \times \frac{1}{5}$				

What have you learned?

Now that you have had some practice using repeated addition to multiply fractions, here is a quicker way to think about this:
Remember: $4 \times \frac{1}{6}$ means the same as $\frac{4 \times 1}{1 \times 6}$

When you multiply fractions you multiply the numerators by each other and the denominators by one another. So the answer is $\frac{4}{6}$.

Time needed
70 minutes

Activity 11:
Multiplying fraction by whole numbers

Work alone

Use what you learnt about rules for multiplying fractions by whole numbers to find quick answers. In some cases you can simplify your answers. In other cases your answer will be a mixed fraction.

1. a) $2 \times \frac{1}{5}$ b) $4 \times \frac{2}{3}$ c) $7 \times \frac{3}{4}$

   d) $9 \times \frac{3}{7}$ e) $11 \times \frac{1}{3}$ f) $8 \times \frac{1}{4}$

   f) $9 \times \frac{2}{3}$ h) $10 \times \frac{2}{5}$ i) $9 \times \frac{2}{3}$

2. Solve these problems:
   a. How long do you do numeracy activities with your children if you have 7 sessions a week and each session lasts $\frac{3}{4}$ of an hour? Give your answer in hours and minutes.
   b. In a recipe you use $\frac{2}{3}$ of a cup of four to make 1cake. How many cups of flour for 4 cakes?
   c. At the party 6 people each ate $\frac{5}{8}$ of a pizza. How many pizzas did they eat altogether?

3. Share your answers with a partner. Make any corrections necessary.

Time needed
25 minutes

Activity 12:
Multiplying fractions by fractions

Work alone

When you halve the recipe below you find the fraction of a fraction. In other words you are multiplying fractions by fractions.

Recipe	**Recipe halved**
$\frac{1}{2}$ cup of oil	$\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$
$\frac{3}{4}$ cup of flour $\frac{2}{3}$ cup of milk $\frac{1}{4}$ cup of sugar one pinch of salt	$\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$ $\frac{1}{2}$ of $\frac{2}{3} = \frac{1}{2}$ $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$ $\frac{1}{2}$ a pinch of salt
For 8 pancakes	**For 4 pancakes**

Find the amount of ingredients needed in each case below:

a) $\frac{1}{3}$ of $\frac{3}{4}$ of a cup flour

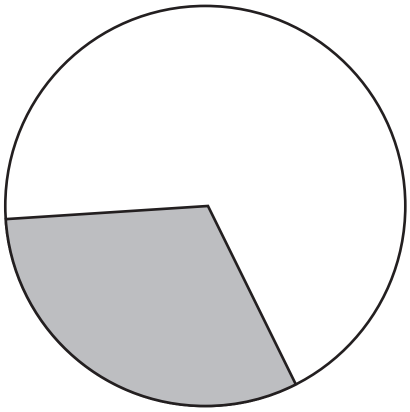
b) $\frac{1}{2}$ of $\frac{1}{8}$ cup of lemon juice

c) $\frac{3}{.5}$ of a litre of milk

d) 3.8 of $\frac{1}{2}$ cup of water

8. Finding fractions of amounts

This pie chart shows that $\frac{1}{3}$ of the people in a certain village is over 80 years old.



Remember $\frac{3}{3}$ or 1 represents the whole population.
There are 960 people in the village

$\frac{1}{3}$ of 960 = $\frac{1}{3} \times \frac{960}{1} = \frac{960}{3} = 320$

So $\frac{2}{3}$ of 960 stands for the population under 80.

Think about how many this is $\frac{2}{3}$ of 960 is.

Trainer's Note:
Discuss the different approaches and ask learners which method they prefer and to say why. Ask them if they have other ways they know to do the same calculation and to demonstrate these to their colleagues.

Phindi thinks in this way:
"If $\frac{1}{3}$ = 320 $\frac{2}{3}$ = 640 (double the number)"

Peter writes it out like this

$\frac{2}{3} \times \frac{960}{1} = \frac{1920}{3} = 320$

Activity 13: Finding fractions of amounts

Time needed
60 minutes

Work alone
Use any of these methods or others that you know, to find these answers:

1. $\frac{3}{4}$ of population of 750 000 people are under 50.
   a. How many people is this?
   b. How many people are over 50?

c. David got $\frac{2}{3}$ of 240 marks in his exam paper correct. What was his score?

d. $\frac{3}{5}$ of the applications for a charity fun run have been received. 2500 applications are still expected. How many have been received?

e. $\frac{5}{8}$ of the 248 children at an ECD Centre are girls.

f. How many is this?

g. How many are boys?

h. $\frac{5}{6}$ of the Lotto prize money of R12 480 000 had been claimed by Monday.

i. How much had been claimed?

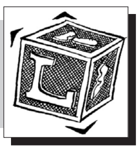
j. What amount had not yet been claimed?

k. One company earns R650 480 per annum. Another earns $2\frac{3}{4}$ times as much. How much does the second company earn?

l. A water tank holds 62 355ℓ. Another holds $3\frac{1}{5}$ times more. How much does the second tank hold?

m. Discuss and compare answers and the methods you used with a partner.

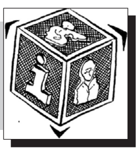
Check your calculations and answers carefully and include this activity in your portfolio for assessment purposes.



What have you learned?

When we see the word of in a number sentence with fractions we should recognise this as a multiplication situation; so $\frac{3}{4}$ of 100 means $\frac{3}{4} \times 100$ which is $\frac{300}{4}$ which is 75.

Once again you will have learned that to do these calculations you need to have a good working knowledge of your multiplication and division facts. You may use different methods to find your answers. For example in the case of question 2 to find $\frac{2}{3}$ of 240 you may have first calculated $\frac{1}{3}$ of 240 as being 80 and then multiplied your answer by 2 to give you 160 or you could have said 240 × 2 is 480 and divided this by 3 to give you 160.



9. Exploring division of fractions

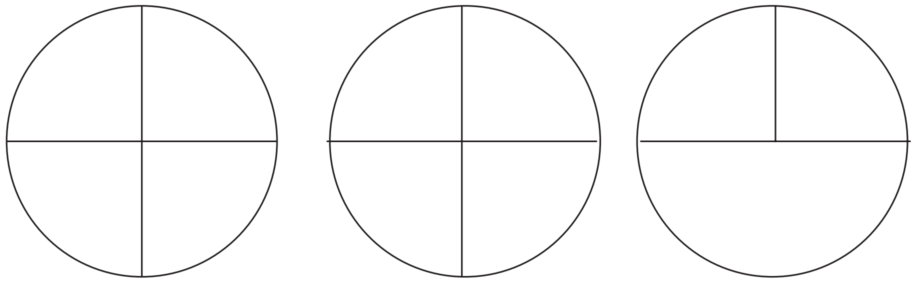
Think about this question:

$2\frac{1}{2}$ pies are divided into $\frac{1}{4}$ s. How many $\frac{1}{4}$ s are there altogether?

This means $2\frac{1}{2} \div \frac{1}{4}$

Divide each share into $\frac{1}{4}$ s

= 10 quarters or $\frac{10}{4}$



You can write a multiplication sentence for the above like this:
$10 \times \frac{1}{4} = 2\frac{1}{2}$

Time needed
20 minutes

Activity 14:
Representing division of fractions

Work alone

Use drawings to help you find these answers. Write a division sentence and a multiplication sentence to match each question and answer.

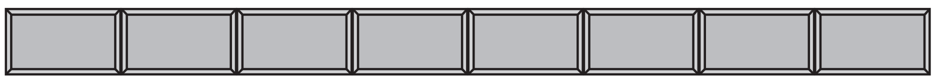
1. Share $2\frac{3}{4}$ pizzas so that each person gets $\frac{1}{4}$. How many people can you feed?
2. 9 people each get $\frac{3}{8}$ of a pizza. How many pizzas do they eat altogether?

Time needed
minutes

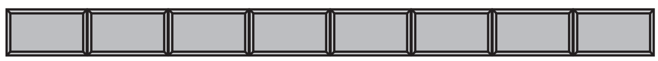
Activity 15:
Multiplication and division of fractions

Work alone

1. A feeding scheme is run at the Bantwana Bami ECD Centre in the winter months. Every morning each child is given a small plate of food. At one time a local company donated a large quantity of chocolate to the Centre, so each child could get a piece of chocolate together with their sandwiches. The chocolate is in bars as shown below:



   a. If there are 150 children, how many bars of chocolate are needed to give $\frac{5}{8}$ of a bar of chocolate to each child?
   b. If there are 20 bars of chocolate, how many children can get $\frac{5}{8}$ of a bar of chocolate each? You can make copies of this drawing to help you.



At another stage, each child gets a small plate of fruit. One each plate there is half an apple, a banana, and three slices of orange.

Each orange is cut into 10 equal slices, so each child gets $\frac{3}{10}$ of an orange. Draw an orange and divide it into 10 equal segments.

c. If only 65 oranges are available, how many children can each get $\frac{3}{10}$ of an orange?

d. How many oranges do you need to give 200 children $\frac{3}{10}$ of an orange each?

e. If each child gets $\frac{3}{8}$ of a bar of chocolate, and there are 120 bars of chocolate, how many children can get chocolate?

This is how Mandisa found her answer to Question 1e.:

1 bar of chocolate is 8 eighths of a bar of chocolate.
So 120 bars is 120 × 8 = 960 eighths.
Each child gets 3 eighths.
So the question is: "How many groups of 3 eighths there are in 960 eighths?"
This is 960 ÷ 3, which is 320.
So 120 children can each be given 38 of a bar of chocolate.

2. Use Mandisa's method to solve the following problems:

a. How many children can each get $\frac{7}{12}$ of a bar of chocolate, if 40 bars of chocolate are available?

b. How many children can each get $\frac{7}{20}$ of a bar of chocolate, if 75 bars of chocolate are available?

c. How many children can each get 4 bars of chocolate, if 275 bars of chocolate are available?

d. What arithmetical operation have you used to solve the problem in question 7?

e. You have R28. How many apples at 64c each can you buy?

3. Compare answers and discuss the methods you used to solve this set of questions Make any corrections necessary.

Time needed
20 minutes

Activity 16:
Division of fractions on a number line

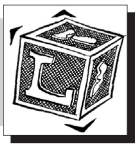
Work alone

Another way to show the process of dividing fractions is to use a number line. On this number line you can see there are 4 quarters in 1 or $1 \div \frac{1}{4} = 4$

1. Use the number line to find quick answers to these.

a) $2\frac{3}{4} \div \frac{1}{4}$	b) $4\frac{1}{2} \div \frac{1}{4}$	c) $3\frac{3}{4} \div \frac{1}{4}$
d) $5 \div \frac{1}{4}$	e) $4\frac{1}{2} \div \frac{1}{2}$	f) $3\frac{1}{2} \div \frac{1}{4}$

2. Write a multiplication sentence to match each example in question 1, for example: $\frac{1}{4} \times 11 = 1\frac{1}{4} = 2\frac{3}{4}$



What have you learned?

Let's look at solutions to some of the work you have done in the Activity 15. You will notice that the following are all division problems.

Question 1b:	$20 \div \frac{5}{8}$
Question 1d:	$200 \div \frac{3}{10}$
Question 2a:	$40 \div \frac{7}{12}$
Question 2b:	$75 \div \frac{7}{20}$
Question 2c:	$275 \div 4$
Question 2e:	R28 ÷ 64c

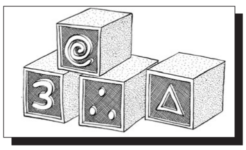
> ***Trainer's Note:***
> *Have a discussion to ensure that learners understand the explanations about the different ways to do these calculations. If learners have additional ideas and thoughts about ways to solve the same problem differently encourage them to share these with their colleagues.*

When you did question 2e, you first changed the R28 to 2800c, and re-wrote the question as 2800c ÷ 64c. In other words, you made the units the same.

This is also what you did in questions 2a and 2b. In question 2a you replaced the 40 whole bars of chocolate with 480 twelfth-bars, to replace the question 40 wholes ÷ 7 twelfths with 480 twelfths ÷ 7 twelfths.

In question 2b you replaced the 75 whole bars of chocolate with 1500 twentieth-bars, to replace the question 75 wholes ÷ 7 twentieths with 1500 twentieths ÷ 7 twentieths.

We can also say that in question 2a you replaced $40 \div \frac{7}{12}$ with $\frac{480}{12} \div \frac{7}{12}$, and in question 2b you replaced $75 \div \frac{7}{20}$ with $\frac{1500}{20} \div \frac{7}{20}$.

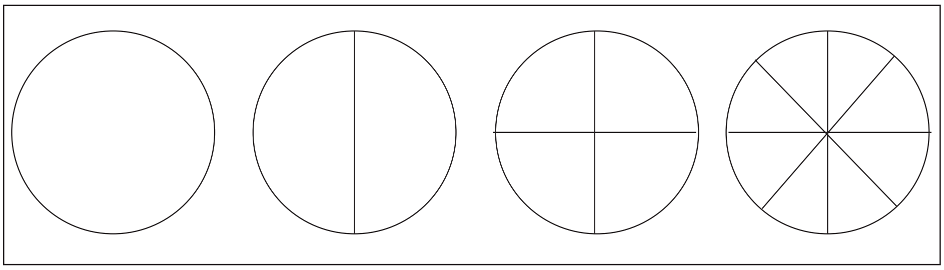


Linking your learning with your ECD work

Young children experience fractions informally when they are sharing. For example they may talk about who got the bigger or smaller piece of cake or more or less sweets or fruit. Try to create opportunities for them to talk about sharing, to use their own language and their own ways to describe their experiences.

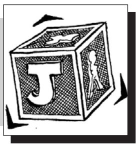
Circle pieces

Some of you will have ready made circles that come as part of a set of shape pieces or part of a fraction set. If not you can make your own. Find a nice big circle that you can cut out of cardboard. Trace and cut out several of these Leave one whole and cut up the others into halves, quarters or eighths.



Play different games like these with the children:

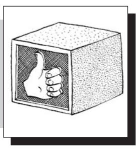
- Mix up the pieces and let them find different ways to make up a whole.
- Tell a story about 2, 4 or 8 people who all shared one pie.
- Let them to use the pieces to show each different ways of sharing the same pie.
- Talk about whose share is biggest, smallest.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about fractions?
b. How do you think you will be able to improve your skills of working with fractions?
c. Write down one or two questions that you still have about fractions.
d. How will you use what you learned about fractions in your everyday life and work?



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Describe different kinds of fractions				
2. Find equivalent fractions by using your knowledge of multiples and factors				
3. Reduce fractions to their simplest form; convert improper fractions to mixed fractions				
4. Do calculations with fractions including addition subtraction, multiplication and division				
5. Solve fraction problems in context, choosing the correct operation to find the answer				

UNIT SEVEN

Investigating Ratio

In this unit you will address the following:

Unit Standard 7449
SO1:
Critically analyze the use of mathematical language and relationships in the workplace. (Wage negotiations, salary increases, and productivity as a ratio.)

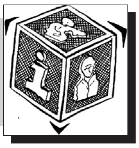
SO2:
Critically analyze the use of mathematical language and relationships in the economy. (Budgeting, banks: interest rates, mortgage, service charges; fuel prices; pensions; inflation; value of the rand and exchange rates.)

SO4:
Critically analyze use of mathematics & mathematical language & relationships in political relations (Income distribution; census; elections; voting; opinion polls.)

Unit Standard 7447
SO3:
Verify and justify solutions within different contexts. (Solutions derived by learners and by others.)

SO7:
Analyze the relationship between rational and whole numbers.

To do this you will
- explain the different meanings of ratio and proportion;
- do calculations in the context of real life situations where ratio and proportion apply, including simplifying ratios, finding equivalent ratios and calculate amounts in the correct proportion according to what is specified;
- distinguish between direct and indirect proportion situations and do related calculations.



1. Ratio

A ratio is a mathematical way of comparing two quantities or amounts that have the same units. In the conversation above Mrs. Maseko and Mrs. Dlamini are comparing the number of people. You cannot for example compare 250g to 1kg but you can find the ratio of 250g to 1000g.

Trainer's Note:
Spend time reviewing the explanation in this text to make sure your learners understand the meaning of ratio and some of the common contexts where we do ratio calculations.

You use a colon sign: to express ratio. You do not write the name of the quantities you are referring to. So in the case above, you could write 1:50 which means one ECD practitioner for every 50 learners. If the text had said that there were 50 learners for every one practitioner however, we would then express the ratio in reverse as 50:1.

There are many examples of ratio in everyday life. In recipes for example you can see:

- 2 cups of water for every cup of rice.
- 3 parts of wheat to1 part of cracked corn to 2 parts of sorghum to make up chicken feed.

Think of more examples where ratio is used to combine quantities and discuss these.

Time needed
10 minutes

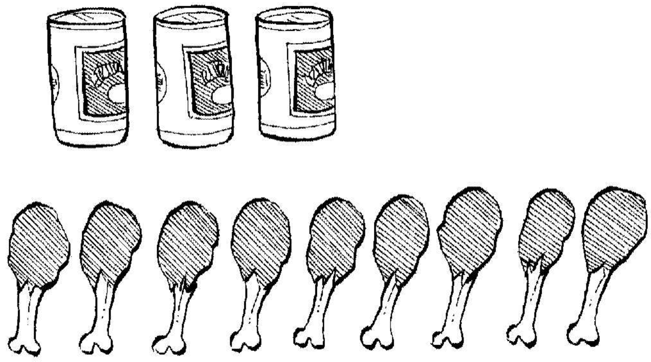
Activity 1:
Comparing Ratios

Work with a partner

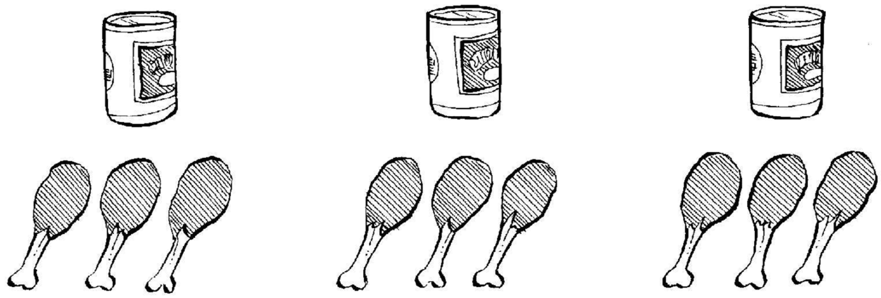
1. Find out the total number of children who attend your ECD site.
   a. Work out the ratio of practitioners to learners.
   b. Work out the ratio of boys to girls.

2. Simplifying ratios

Ratios are usually written in their simplest form. In this chicken advert for example, the family pack consists of 3 coldrinks and 9 drumsticks. The ratio of cold drinks to drumsticks is 3:9.



We can regroup the cold drinks and the drumsticks to show that the simplest form of 3:9 is 1:3. So that means for every 1 cold drink here are 3 drumsticks. The simplified ratio of 3:9 is 1:3. In a picture it would like this:



- To simplify a given ratio you simply divide each part by the same number. So 4:8 = 1:4 (divide each part by 4) or 12:36 is 1:3 (divide each part by 12).
- To express a ratio in its simplest form we reduce the ratio to the lowest natural number that both quantities divide into.

Activity 2:
Simplifying ratios

Time needed 15 minutes

Work alone

1. Use a colon to express these amounts as ratios. Then write each ratio in their simplest form.

   a) R2 to R4
   b) 6,3cm to 9,3cm
   c) 10 educators to 350 learners
   d) 3 cups of rice to 6 cups of water
   e) 39 girls to 13 boy
   f) 17 girls to 13 boys
   g) 4.5 litres of water to 1,5 litres of paint
   h) 35 Pens to 45 pencils

What have you learned?

From simplifying ratios in the previous activity you found out for example that R2 to R4. Expressed as a ratio is written as 2:4, which simplified is 1:2, or that 39 girls to 13 boys expressed as a ratio is 39:13 which simplified is written as 3:1. You call these ratios equivalent ratios.

> ***Trainer's Note:***
> *Make time for learners to review answers. Give them additional examples if necessary. As in the case of fractions, they need to know their multiplication and division facts to be able to do these calculations.*

Activity 3: Equivalent ratios

Work alone

Fill in the missing numbers to make these ratios equivalent. Explain your method to a partner.

a) 3 : 2 = 9 : ?　　b) 5 : 3 = 20 : ?　　c) ? : 5 = 8 : 20

d) 15 : ? = 3 : 4　　e) ? : 3 : ? = 30 : 18 : 24　　f) ? : 14 = 14 : 28

g) 3 : 4 : 7 = 6 : 8 : ?　　h) 500: ? : ? = 25 : 5 : 125

What have you learned?

Equivalent ratios can be found by multiplying both numbers by the same factor so 3:2 is the same as 9:6. You do this by multiplying the both values by 3.

3. Converting

Remember we said that to compare quantities in a ratio, they must be the same unit of measure. If they are not the same you need to convert (change) them into the same unit of measure. So if you want to write the ratio of 600m to 2km in the simplest form you need to do the following conversion:

2km = 2 × 1 000m = 2 000m so 600m to 2 000m = 600:2 000 = 6:20 = 3:10

Activity 4: Equivalent ratios

Time needed 10 minutes

Work alone

Write these ratios in their simplest form

a) 1m to 60cm　b) 40cm to 2m　c) 1,2 hours to 45 minutes　d) $2\frac{1}{2}$ hrs to $\frac{1}{4}$ hr

What have you learned?

When comparing ratios you have to convert them to the same units. So 1m to 60cm when compared as a ratio would be expressed as 100:60 simplified to 10:6 or 5:3. Sometimes it is easier to convert to smaller units first as in d) above, like this: $2\frac{1}{2}$ hours to $\frac{1}{4}$ hour = 150 minutes to 15 minutes = 150:15 = 15:1,5.

4. Using ratio to calculate information

Here is a plan of three tables in one of the classrooms at the Bantwana Bami ECD Centre. B stands for boys and G stands for girls.

B	G	G	B	G	G	B	G	G
	G	G		G	G		G	G

The ratio of boys to girls in this classroom is 1:4. So for every one boy there are four girls.

So you can say that $\frac{1}{5}$ of the children are boys and $\frac{4}{5}$ of the children are girls. So if you know for example there are 40 children in the group, you can find out how many are girls and how many are boys:

$\frac{1}{5}$ of 40 = 8 boys

and if $\frac{1}{5}$ is 8 then $\frac{4}{5} = 8 \times 4 = 32$ girls

or $\frac{4}{5}$ of 40 = (40 – 8) = 32 girls

Activity 5:
Solving Ratio Problems

Time needed
10 minutes

Work alone

Follow the example above to find these amounts:

In Mrs. Maseko's class of 3 – 4 year olds, there is a ratio of 3 girls for every 2 boys.

1. What fraction of the children are girls?
2. What fraction are boys?
3. There are 35 children in her class. How many are boys? How many are girls?

5. Dividing in ratios

To divide an amount of R40 in the ratio of 3:5 you say:
3:5 is 8 parts altogether.

$\frac{3}{8}$ of R40 = $\frac{3}{8} \times \frac{40}{1}$ = R15

$\frac{5}{8}$ of R40 = $\frac{5}{8} \times \frac{40}{1}$ = R25

Check: R15 + R25 = R40
So R40 in the ratio of 3:5 is R15:R25

Trainer's Note:
Make time to discuss and review answers. Give more practice examples if necessary or have learners make up their own examples to ask one another.

Activity 6:
Dividing amounts in given ratios

Work alone
Divide each amount in the given ratio:
a) R200 in the ratio of 3:2 b) R888 in the ratio of 5:3 c) 50kg in the ratio of 3:7

6. Percentages and ratio

We can also express ratios as percentages. For example Activity 5 the girls made up $\frac{3}{5}$ of the class and the boys $\frac{2}{5}$.

$\frac{1}{5}$ of 100% = 20%. So $\frac{3}{5}$ = 60% and $\frac{2}{5}$ = 40%.

Activity 7:
Percentages and Ratio

Time needed
5 minutes

Work alone
Read the example on page 70 under "Using ratio to calculate information" again. Calculate the percentage of boys to girls.

What have you learned?

Ratios can be expressed as fractions or percentages. So if 1:4 represents the ratio of boys to girls in a group of 5 children, another way of expressing this would be to say that $\frac{1}{5}$ are boys and $\frac{4}{5}$ are girls, or another way of saying this would be that 20% are boys and 80% are girls.

> ***Trainer's Note:***
> *The terms ratio and proportion are often used interchangeably. Make sure that their meanings are clearly understood. You can use the following example to demonstrate what we mean by either term and to see how they relate to one another.*
> *Proportion: If a model of a house is one fifth of the actual size, the model of the door of the house must be one fifth of the size of the real door.*
> *So we can say that the model and all its parts are in proportion. Or in the case of numbers, we could say that the numbers 4 and 16 are in the same proportion as the numbers 1 and 4 because 4 is $\frac{1}{4}$ of 16 and 1 is $\frac{1}{4}$ of 4.*
> *Ratio: If a model of a house is one fifth of the actual house the length of the model is one fifth of the length of the actual size of the house. We could say that their lengths are in the ratio of one to five which we would then write as 1:5.*

7. Ratio and Proportion

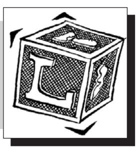
When you follow a recipe you work with ratio and proportion. If two ratios are equal then they are in proportion.
For example:
You need 2 cups of water for every cup of rice you cook. You put $6\frac{1}{2}$ cups of rice in a pot.

a. How many cups of water should you add to the rice?
b. If a cup is about 250ml, how many ml of water is this?
c. Express as a ratio the number of cups of water you will need to cook $7\frac{1}{2}$ cups of rice.

Activity 8:
Ratio and proportion

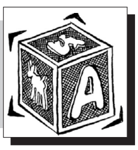
Work alone

1. Mrs. Maseko has to mix one litre of water-based paint with $3\frac{1}{2}$ litres of water to get a good paint mixture. How much water must she add to make a mixture if she uses three times as much paint?
2. To make Mary's Chicken feed mixture you need 3 parts of wheat to 1 part cracked corn to 2 parts of sorghum.
   a. Mary uses 750g of wheat. How much cracked sorghum should she use?
   b. What is the final mass of the mixture in kilograms?
   c. Mary needs to mix enough chicken feed for a week. The chickens eat 600g of feed every day. How much wheat and how much sorghum must she put into the mixture?
3. Share your thinking and your calculations for working out answers questions 1 - 3. Evaluate which methods are easiest to follow. Make any corrections you need to.



What have you learned?

Recipes are often given in ratios and if you increase one part of the recipe you need to increase the other parts in equal proportions to get the right "mix".



Time needed 30 minutes

Activity 9:
Comparing direct and indirect proportion

Work alone

This table shows the cost of buying different quantities of cement per bag.

1. Try to find quick ways to work out the answers. Fill in your answers.

No of bags of cement	1	10	20	40	50	75	100	125
Cost in Rand	45	450						

Work with a partner

2. Compare answers with a partner. Discuss and evaluate the different methods you may have used to find quick answers.
3. Was the price of cement reduced if you bought more quantities? Explain how you know.

What have you learned?

You will have noticed that the cost of 1 bag was R45 and the cost of 10 bags R450. If you simplify the ratio of 10:450 you get back to the cost ratio of 1:45 which is the cost per bag of cement. If you simplify the ratio for the other amounts you will find that they too are in the ratio of 1:45. This shows that there was no discount offered for more bags you bought. The cost remains at R45 a bag, whatever the quantity. You may have used different quick methods to find the missing values in the table. Here is one method:
Find the cost of 100 bags, which is R45 × 100 = R4 500. Halve this amount to get to the cost of 50 bags which is R1 250.

Trainer's Note:
Assess learners' answers and discussions and consider if you need to give additional practice activities or whether to move on.

8. About direct and indirect proportion.

There are two kinds of proportion. The cost of the bags above is an example of direct proportion. As the number of bags increased, so did the cost. You can say that these quantities, the cost and the number of bags of cement, were in direct proportion to each other. If the number of bags doubled or trebled, so did the cost. So the cost of the cement was directly proportional to the number of bags of cement bought.

Trainer's Note:
Read through this text together with your learners to make sure they understand the differences between the concepts of direct and indirect proportion and the kinds of situation where either type applies.

The other kind of proportion is called indirect proportion. This means that as one amount increases the other decreases. Or as one amount decreases the other increases. Here is an example to explain how this works in practice:

6 builders take 24 days to finish building the new library at the Bantwana Bami ECD. Think about how long it will take 12 builders to do the same job. What about 3 builders? If you double the number of workers it should take half the time! If you halve the number of workers it will take double the length of time

Activity 10:
Working with Indirect proportion

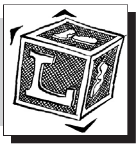
Time needed 20 minutes

Work alone

1. Use your own methods to find out how long the library building job will take if the builders all work at the same rate and there are:

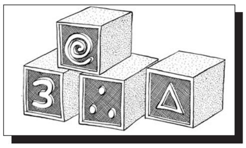
   a) 1 builder b) 8 builders c) 24 builders d) 20 builders e) 4 builders

2. With a partner discuss and compare the thinking and the methods you used to work these out.



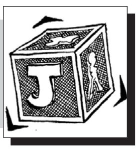
What have you learned?

In this example you investigated indirect proportion. Where you increased one quantity (the number of builders) you cause a decrease at the same rate in the other quantity (time taken) or visa versa. You say that the two quantities, are indirectly proportional. So the more workers, the less time it would take, and the fewer workers the more time the job would take.



Linking your learning with your ECD work

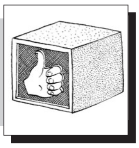
- Pour enough juice concentrate into a glass and discuss how much water you need to add to make the right solution.
- Use a bigger glass and add twice as much concentrate and ask the children how much water you need to add this time to make the right solution. Encourage them to explain why the more concentrate you add, the more water you need to add.
- Children can find out how much white paint they need to add to red paint to produce a pink paint. What happens if you double or treble the amount of red?



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about ratio?
b. How do you think you will be able to improve your understanding of ratio?
c. Write down one or two questions that you still have about percentages and ratio.
d. Write down one or two questions that you still have about ratio and proportion.
e. How will you use what you learned about ratio, percentages and proportion in your every day life and work?



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Explain the different meanings of ratio and proportion				
2. Do calculations in the context of real life situations where ratio and proportion apply, including simplifying ratios, finding equivalent ratios and calculate amounts in the correct proportion according to what is specified				
3. Distinguish between direct and indirect proportion situations and do related calculations				

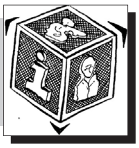
UNIT EIGHT

Calculating with decimals

In this unit you will address the following:

Unit Standard 7447
SO7:
Analyze the relationship between rational and whole numbers.

To do this you will:
- name the value of each digit in a decimal number;
- round off decimals to the nearest whole numbers, tenths, hundredths and thousandths;
- use rounding off strategies to estimate answers when calculating with decimals;
- apply knowledge of the decimal system to solving problems using decimal measures;
- do calculations with division and multiplication of tenths and hundredths and find and explain links between the answers your get;
- extend your number skills and methods of calculating to working with decimal values;
- use your calculator as a checking tool to check answers to calculations involving decimals.



1. More about decimals

Decimal numbers are those numbers in our base ten number system that represent values less than one. Just like whole numbers, they are grouped in "bundles of ten" - in this case tenths, hundredths, thousandths and so on. Like whole numbers, decimal numbers are infinite and it is possible to get a number that is on billionth or one trillionth! But of course we seldom work with such small numbers in everyday life.

Trainer's Note:
It is important that you help learners to see that decimal numbers are a continuation of our base ten numbers system and the relationships that exist between each value in the system i.e. ten times bigger or smaller than the unit to its left or right.

Note that in some countries they use a full stop to show the value of decimal numbers but the convention we follow is to rather use a comma except with money. Also when naming decimal numbers we do not for example say one comma thirty four, but rather one comma three four.

In this place value chart below we have broken down the number 15 873,5679 to show the value of each digit in the number from ten thousands to ten thousandths.

Ten thousands	Thousands	Hundreds	Tens	Units		Tenths	hundredths	thousandths	ten thousandths
1	5	8	7	3	,	5	6	7	9

Or we could write it out in expanded form like this:
$10\,000 + 5\,000 + 800 + 70 + 3 + \frac{5}{10} + \frac{6}{100} + \frac{7}{1000} + \frac{9}{10\,000}$

Time needed
20 minutes

Activity 1:
More about decimals

Work alone

1. Write each of these decimal numbers in expanded form. Also practice saying the number out loud to yourself.

   e.g. 4,73 = $4\frac{7}{10} + \frac{3}{100}$ (four comma seven three)

   a) 16,056   b) 109,78   c) 999,999   d) 567,032   e) 78,009

2. Write the value of each digit in these numbers
   e.g. 56,78 = five tens, six hundreds, seven tenths, 8 hundredths

   a) 45,8972   b) 234 567,092   c) 78 960 709,0987   d) 863,9901

3. Compare answers with a partner. Make any corrections necessary.

Time needed
20 minutes

Activity 2:
Ordering decimals on a number line

Work Alone

You can order decimal numbers in a number sequence like this to help you count on in tenths, hundredths and thousandths.

For example these are the decimal hundredths that fall between 0,1 and 0,2.

0,1   0,11   0,12   0,13   0,14   0,15   0,16   0,17   0,18   0,19   0,2

Count on in decimal fractions to fill in the missing decimals in these sequences

a) 0,2   0,21   -   -   -   -   -   -   -   0,3

b) 0,01;   0,011   -   -   -   -   -   -   -   -   0,02

c) 0,1   0,2   -   -   -   -   -   -   -   -   -   1,2

1. Draw your own number sequences to show the position of the numbers between:

   (i) 4,001 and 4,01   (ii) 4,311 and 4,321   (iii) 7,5 and 7,65   (iv) 3,002 and 3,003

Work with a partner

2. Discuss and compare your answers with a partner. Make any corrections necessary.

> ***Trainer's Note:***
> *To reinforce what the value of each digit in these numbers is, first have them practise saying each number in the sequence out loud; e.g., zero, comma one, zero comma one one etc. and then to name the value of each digit in the number. So in the first number the 1 in 0,1 stands for one tenth and in the number 0,11 the first digit stands for tenths and the second for hundredths.*

2. Using decimals as a classification system.

The decimal system is used as an ordering system in libraries to classify books of different kinds. Non-fiction books are numbered according to the subject, while authors' names are arranged in alphabetical order. This system was named after its inventor, an American librarian called Melvil Dewey in the 1870s. Note the Dewey system uses a point rather than a decimal comma. We also use a point when we refer to money. The use of decimals here does not show which value is larger or smaller than another, but rather refers to the order in which the books are classified and should appear on the library shelves.

Time needed
10 minutes

Activity 3:
Classifying books

Work alone

A book about religion is classified by the number 231.63.

1. Fill in the five numbers that come before this book.

   (i) (ii) (iii) (iv) (v)

2. The following are the only five books still on the literature shelf. The rest have been taken out. Put these books in order from first to last, starting with the smallest number.

   331.3; 331.36; 331.08; 331.91; 331.065

Work with a partner

3. Discuss and compare your answers. Make any corrections necessary.

3. Rounding off decimals

Trainer's Note:
Your learners should have prior experience rounding off whole numbers to the nearest tens, hundreds place etc to do this section with ease. Give practice doing this if necessary.

Just as with whole numbers, both for estimation purposes and when doing calculations with measurements or money, we often find ourselves having to round off decimal values. For example, you go shopping and do an estimation of your purchases as you go along. Let's say you have bought two things so far for R4.65 and R19.35. You might round off these amounts in our head to R5.00 + R19.50 which is R24.50.

Once you learn the steps to follow to round a decimal number to the nearest $\frac{1}{10}$ or $\frac{1}{100}$, you can then use what you learned to round decimal numbers off to the nearest whole number and the nearest thousandth.

Rounding to tenths

If you wanted to round the number 3,67 to the nearest $\frac{1}{10}$, you look at the value of the $\frac{1}{10}$ place. If it is 5 or more, you round up to the next $\frac{1}{10}$. If it is less than 5 you round down; so 3,67 becomes 3,7 but a number like 3,44 would be rounded down to 3,4.

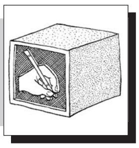
Rounding to hundredths

To round off values to the nearest $\frac{1}{100}$, you need to look at the value of the place $\frac{1}{1000}$. So 6,432 becomes 6.43 but 6,439 is rounded down to 6.44.

Time needed 30 minutes

Activity 4: Rounding off decimals

Once you have checked your answers add this activity showing all your calculations to your portfolio.



Work alone

1. Practice rounding these numbers or measurements to the nearest $\frac{1}{10}$

   a) 4,08m b) 15,72cm c) 123,08kml d) 6,0001g

2. Practice rounding these measurements to the nearest $\frac{1}{100}$

   a) 5,348g b) 3,322ml c) 17,009kg d)17,382ℓ

3. Round the same measurements in 2 to:

   a) the nearest whole number b) the nearest $\frac{1}{0}$

4. Now use what you found out to round these decimal numbers to the nearest whole gram to estimate the total mass of these amounts.

   a) 123,098kg + 43,7kg + 45,2kg + 35,98kg b) 34,76mℓ + 34,892mℓ + 40,08mℓ

5. Use a calculator to check how close your estimates were in each case.

6. Work out a rule for rounding off the nearest $\frac{1}{100}^{th}$ and then round these values off to the nearest $\frac{1}{100}$

   a) 14,5467 b) 65,98 760 c) 23,5611 d) 4,8731

7. Which numbers are closest to 6,001?

   6; 6,1 ; 7; 6,002

8. Which numbers are closer 17,09?

   17; 17,1; 18; 17,2

9. Discuss and compare answers with a partner.

Time needed
30 minutes

Trainer's Note:
Do this investigation together first. Learners can first read the pattern on the page and then check it using their calculators.

Activity 5: Multiplying and dividing decimals

Work alone

Let's look at what happens when we multiply decimals in this place value chart.

4,6 × 10 = 46
4,6 × 1 00 = 460
4,6 × 1 000 = 4 600
4,6 × 10 000 = 46 000

1. Discuss the patterns and see if you can explain the pattern and what is happening to the number each time.
2. Now multiply each of the numbers below first without using your calculator. Then check to see if you were right using your calculator. Look for patterns to try and help you understand what is happening to the numbers in each case. Share your findings with a partner.

a) 3,6 × 10	b) 3,6 × 100	c) 3,6 × 1 000	d) 3,6 × 10 000
e) 9,45 × 10	f) 9,45 × 100	g) 9,45 × 1 000	h) 9,45 × 10 000
i) 438,087 × 10	j) 438,087 × 100	k) 438,087 × 1 000	l) 438,087 × 10 000

Stop and Think

- Thami says when you multiply a decimal number by 10 the decimal comma moves one place to the right.
- Mabel says when you multiply a decimal number by 10, the digits shift one place to the right.
- Who do you think is right? Discuss your ideas with a partner

3. Can you predict what will happen when you divide a decimal number by 10 or 100 or 1000? Try these using your calculator. Look for patterns to help you explain the answers you get. Share your findings with a partner.

a) R166,55 ÷ 10	b) R166,55 ÷ 100
c) R2598,05 ÷ 10	d R2598,05 ÷ 100

4. Use what you found out to predict the answer you will get when you divide each of the amounts of money above by 1 000. Use your calculator to check if your predictions were correct. Share your findings with a partner.

Activity 6 :
Multiplying numbers by tenths and hundredths

Time needed
30 minutes

Work alone

1. Investigate what happens when you do these calculations with your calculator

a) 13 ÷ 10	b) 13 × 0,1
c) 65 ÷ 10	d) 65 × 0,1
e) 34 897 ÷ 10	f) 34 897 × 0,1

Stop and Think

You probably found out that multiplying a number by 0,1 is the same as dividing that number by 10. Can you explain why? Share your findings with a partner.

2. Now investigate what happens when you divide the same number by 100 and multiply it by 0,01. Use your calculator to find these answers.

a) 42 × 0,01	b) 42 ÷ 100
c) 352 × 0,01	d) 352 ÷ 100
e) 678 × 0,01	f) 678 ÷ 100

3. Write a multiplication number sentence that is the same as:

a) 657 ÷ 100 b) 8 701 ÷ 100 c) 6 ÷ 100 d) 17 ÷ 100

4. Write a division number sentence that is the same as:

a) 4 × 0,1 b) 43 × 0,01 c) 117 × 0,1 d) 59 × 0,01

Stop and Think

You now found out that multiplying a number by 0,01 is the same as dividing that number by 100. Can you explain why? Share your findings with a partner.

Activity 7:
Dividing numbers by tenths hundredths and thousandths

Time needed
30 minutes

Work alone

1. Use your calculator to find quick answers to these:

a) 41,5 ÷ 0,1	b) 41,5 × 10
c) 102,04 ÷ 0,1	d) 102,04 × 10
e) 347 ÷ 0,1	f) 347 × 10

2. Discuss and share findings with a partner.

3. Use a calculator to find these answers:

Trainer's Note:
Discuss and review answers together. Have learners explain the relationships between the two sets of numbers for example, and why in an example like 43,01 ÷ 0,01 and 43,01 × 100, the answer is the same. i.e 4301. Let them make up some of their own examples where a multiplication and division calculation will yield the same answer.

a) 123,01 ÷ 0,01 b) 123,01 × 100
c) 43,01 ÷ 0,01 d) 43,01 × 100
e) 560 ÷ 0,01 f) 56 × 100

4. Write another rule to explain your answers to 4.
5. Write a multiplication number sentence to match each of these. Use your calculator to check if you were correct.

   a) 7, 0 ÷ 01 b) 0,04 ÷ 0,1 c) 76,35 ÷ 0,01 d) 4,34 ÷ 0,01

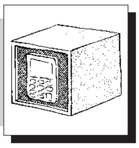
Time needed
30 minutes

Activity 8:
Multiplying decimals by larger multiples of 10

Work alone
Floor boards come in lengths of 4m. The carpenters who are fitting a floor in a shopping mall work out they need 400 floorboards to cover the floor of a workshop. The lengths of board have to be to be 343,7cm long. Tabitha is a quantity surveyor working on the job. She has left her calculator at home so she does this calculation to find out the total length of floor boards that she needs.

- She sees 400 as 100 × 4 so she first multiplies the length of one board by 100 to get
  343,7cm × 100 = 34 370cm
- She then needs to multiply this amount by 4 (to make 400)
- So she first doubles the amount to get to 68 740cm
- Then she doubles this answer again to get to 137 480cm
- Lastly she changes this into metres by dividing the length in cm by 100 to find
  how many metres this is (because there are 100cm in a m)
- This gives her 1 374,8m. She rounds this off to 1 375m.

1. Use your knowledge of metric units, and Tabitha's method to find these amounts. Round your answer off to the nearest whole unit in each case.
   a) 456,3 litres × 800 b) 703,12m × 200
   c) 67,875kg × 600
2. Discuss and compare your answers with a partner. Use your calculator only as a checking tool.
3. Gideon does a quick calculation for 0,6384m × 700 like this:
   0,6384m × 10 000 = 6384m
   (This helps him to get rid of the decimal comma for the moment)
   6 384 × 7 = 44 688m
   But he only needed to multiply by 100, not 10 000 so he divides the answer by 100 to get
   44 688 ÷ 100 = 446,88m



4. Use a calculator to check if Gideon's method works and his answer is correct.
5. Think about how and why Gideon's method works and use the same approach to find quick answers to these:

   a) 4,2896 × 30 000   b) 707,2 × 600   c) 0,5232 × 8 000

6. Discuss and compare answers. Use your calculator only as a checking tool.

Time needed
30 minutes

Activity 9:
A quick way to multiply decimals by big numbers

Work alone
Now that you have worked hard using written ways to multiply decimals by big numbers, here is a way to help you think about this more easily!

e.g. 143 × 21,4

- There is one digit after the decimal comma in one of the numbers, so that means there must be the same number of digits after the decimal comma in your answer.
- Do a multiplication sum as though you were working with whole numbers.
- 143 × 214 = 30 602
- Now divide by 10 again to get to your answer of 3060,2 - because one of your original numbers had decimals tenths so you need to divide your first answer by 10.

1. Use this thinking to find answers to these. Only use a calculator as a checking tool.
   How much is?
   a) 36 cups of juice if there are 45,2ml in each cup.
   b) 780kg of meat @ R5,85 per kg
   c) 35 taxi trips, where each trip is 56,8km long
   d) 78,45 metres of wood @ R85 per m

Pule discovered the same principle works where there is more than one decimal number in the calculation. So when he does a calculation like 1,2 × 3,5, he changes this to 12 × 33 = 396 . He knows that he must then have two numbers after the decimal comma because both numbers were written with decimal tenths. So he divides the first answer he gets by 100 to write his answer as 3,96.

2. Apply Pule's thinking to find these measurements:
   a. The area of a rectangle with sides 3,04m by 14,53m
   b. The volume of a cube with sides 62,4cm; 14,32cm ; 17cm
   c. Check and compare answers with a partner.

Stop and think

How do you think you would adapt this thinking to solve problems where you have to divide a decimal number or measurement instead of multiplying?

3. Try it out with these examples. Use a calculator to check your answers. Share your findings with a partner.

   a) 200,35 ÷ 5 b) 189,27 ÷ 9 c) 60,006 ÷ 6 d) 5649 ÷ 7

4. Dividing decimals by multiples of 10

Dividing decimals by multiples of 10 is easy. Look at this example

4, 65 ÷ 10 = 0,465

The numbers move one place to the right of the decimal comma as they get ten times smaller. So if you have to calculate 4,65 ÷ 50, you can first divide by 10 = 0,465 and then by 5 = 0,093. If the number does not divide exactly, round your answers off to the nearest $\frac{1}{100}$.

Time needed
20 minutes

Activity 10:
Dividing decimals by multiples of 10

Work with a partner

1. Follow this method to find quick answers to these. Only use your calculator as a checking tool afterwards.

   a) 9,6 ÷ 300 b) 147,62 ÷ 700 c) 19,09 ÷ 400 d) 78,98 ÷ 500

2. Which answer is correct for this sum: 41,7893 ÷ 1 000 = ?

   a) 0,417893 b) 4178,93 c) 0,0417893 d) 417,893

Time needed
20 minutes

Activity 11:
Dividing decimals by decimals

Work with a partner

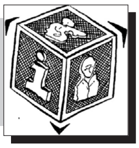
To divide a decimal by a decimal (e.g. 12,036 ÷ 0,05), you can use a calculator or follow this written method:

- Change your divisor (the number you are dividing by) to a whole number. So 0,05 becomes 5. In this case you multiply by a 100.
- Multiply the dividend (number you are dividing into) by the same amount. So 12,036 becomes 1203,6.

- Divide 1203,6 by 5 (add a 0 after the 6 which gives you 1203,60)
  Note: We do this so that the number is divisible by 5. Remember that by adding a 0 to the last digit of a decimal number, you do not alter its value.
- Answer is 240,72.

1. Try the method with these examples. Round your answers off to the first decimal place where necessary. Only use your calculator as a checking tool.

   a) 22,6 ÷ 0,03 b) 14,2 ÷ 0,004 c) 1,48 ÷ 0,012 d) 0,16 ÷ 0, 06



5. Adding and subtracting decimals

You can use different methods to add decimals. Add values together that are the same. Do not confuse values. If you choose to use the vertical format remember to line up the digits underneath each other.

Adding decimals using the vertical method

Carry over $\frac{1}{10}$ Carry over $\frac{1}{100}$

$$\begin{array}{r} 4,\ {}^{1}3\ {}^{1}8\ 2 \\ +5,\ 2\ 7\ 9 \\ \hline 9,\ 6\ 6\ 1 \end{array}$$

Subtracting decimals using the vertical method

$$\begin{array}{r} 37,436 \\ -\ 42,353 \\ \hline 79,789 \end{array}$$

or in the case where you have to rename or exchange values like this:

Rename as $\frac{16}{1000}$

Rename as $\frac{17}{100}$

Rename as $\frac{7}{10}$

$$\begin{array}{r} 1\ 7,\ {}^{7}8\ {}^{17}8\ 6 \\ -\ 3,\ 4\ 8\ 9 \\ \hline 1\ 4,\ 3\ 9\ 7 \end{array}$$

With decimal numbers the zeroes after the last digit in a number have no meaning but when you add or subtract decimals, it is useful to write them in. So for example, 6,91 – 0,7 is easier to calculate if you add a zero after the 7 so that both numbers have the same number of digits. i.e. 6, 91 - 0,70

Time needed
30 minutes

Activity 12:
Adding and subtracting decimals

Work alone

1. Use rounding off to estimate the total of these amounts. Next do a calculation using any method of your choice to find the actual answer. Use a calculator to check your answers and to see how close your estimations were.

   a. A courier company's minibuses travel these distances in one day. How far is this altogether?

      178,9km + 81,72km + 19,676km+ 342,009km + 100,029km

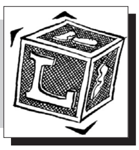
   b. A clothing shop records these purchases for a Monday morning. How much is in total?
      R 43,95 + R675,20 + R56,05 + R1 000,10 + R345,65 + R1 209,10

   c. If there sales for the previous Saturday were R5 550.75. How much less are their sales for Monday ?
   d. A bus has 45,21km left to travel of a 156,65km journey. How far has the bus already travelled?
   e. In gymnastics competition two athletes got these scores (out of 10) for 5 events. The scores were rounded off to the nearest hundredth place:
      Athlete A: 9,01; 7,5; 6,79; 8,07; 7,45
      Athlete B: 9,56; 7,86; 5,43 ; 6,01; 5,49

   f. Order the scores of each athlete from fastest to slowest.
   g. Find the difference between the highest and lowest of all these scores.

Work with a partner

2. Discuss and compare answers. Make any corrections necessary.

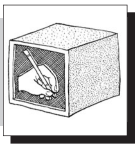


What have you learned?

You should now feel more comfortable to work with decimal numbers maybe you found ways of using what you knew before about calculating with whole numbers, to do similar calculations involving decimals. Some of the examples you have worked with include working with decimals in the context of metric units, because this is where they are most commonly used.

We all know that you can solve any problem using a calculator. But being able to do written calculations using a variety of strategies, builds one's number knowledge and confidence to think mathematically. In the last activity you will have more practice doing calculations with decimals in the context of everyday kinds of problems, involving measurements of different kinds.

Time needed
30 minutes



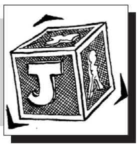
Activity 13:
Using decimals to solve everyday problems

Add the work done on this activity to your portfolio.

Work alone

Solve these problems by using some of the strategies you learnt from doing the activities in this unit. Only use your calculator as a checking tool. When you are finished compare answers and calculation methods with a partner.

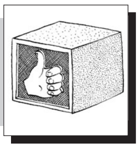
1. 14 households in a rural Limpopo community are given a total area of 16,43 square kilometres in a land claim settlement.
   a. If the land is to be shared equally among all 14 households. How much land does each household get?
   b. If the government pays R12 500 per square kilometre how much money does the whole piece of land cost them?
2. A local council calculated on average that each household under their jurisdiction uses 17,345kl of water per month.
   a. How much water do 630 households use?
   b. If water costs R0.95c per litre. how much do the 630 households pay for their water altogether?
   c. What is the income the council receives for all the water used by the 630 households?
3. A bottle of pills contains 300 pills. Each weighs 0,285g. What is total mass of the bottle's contents?
4. The instructions on a packet of fertilizer say that you must use approximately 0,65kg per square metre of lawn. The area that the Bantwana Bami ECD Centre use as lawn covers 1 750 square metres. How much fertilizer do they need to use?
5. Dumisani buys 300g of cheese that costs 35,40 a kilogram. How much does he pay?
6. Arrange these times clocked by 10 young runners in the 100 metre sprint from fastest to slowest: 14,06; 15,98; 15; 19; 3,89; 14,99; 15,98; 16,01;15,57; 14,89; 15,09
7. How many decimal places will there be in each of these answers?
   a) $56{,}098 \times 34{,}5$ b) $12{,}001 \times 0{,}4$ c) $67{,}904 \times 6$
8. Bongi's car travels 45,35 kilometres on four litres of petrol How far does it travel on one litre of petrol?
9. Use a calculator to check your answers. Discuss and compare calculation methods with a partner.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about decimals?
b. How do you think you will be able to improve your skills of working with decimals?
c. Write down one or two questions that you still have about decimals.
d. How will you use what you learned about decimals in your everyday life and work?



Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Name the value of each digit in a decimal number;				
2. Round off decimals to the nearest whole numbers, tenths, hundredths and thousandths;				
3. Use rounding off strategies to estimate answers when calculating with decimals ;				
4. Apply knowledge of the decimal system to solving problems using decimal measures;				
5. Do calculations with division and multiplication of tenths and hundredths and find and explain links between the answers your get.				
6. Extend your number skills and methods of calculating to working with decimal values;				
7. Use your calculator as a checking tool to check answers to calculations involving decimals.				

UNIT NINE

Moving between percentages, fractions and decimal fractions

In this unit you will address the following:

Unit Standard 7449
SO1:
Critically analyze the use of mathematical language and relationships in the workplace. (Wage negotiations, salary increases, and productivity as a ratio.)

SO4:
Critically analyze use of mathematics & mathematical language & relationships in political relations (Income distribution; census; elections; voting; opinion polls.)

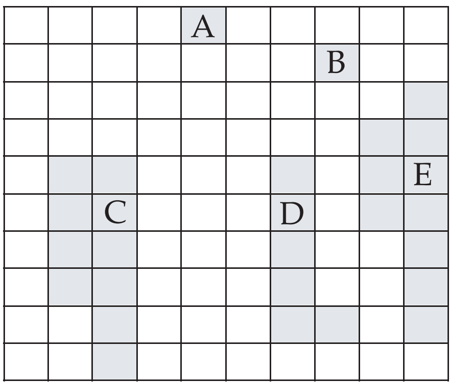
Unit Standard 7447
SO7:
Analyze the relationship between rational and whole numbers.

To do this you will
- explain the meaning of the term percentage;
- name percentage amounts that are both shaded and un-shaded in different square grids;
- express percentage amounts as common fractions - and the other way round;
- express decimal amounts as percentages - and the other way around;
- solve real world problems where you have to calculate percentage amounts;
- use the correct keys on your calculator to do percentage calculations and to check your manual calculations of percentages;
- investigate and explain the value and meaning of different recurring decimals.

1. Percentages

A percentage is another kind of rational number. $\frac{1}{100}$ means on hundredth or one per hundred. Another name for $\frac{1}{100}$ is one percent, written as 1%. Percent comes from the Latin word centum meaning one hundred. Percentages compare quantities as different parts out of 100.

One way of visualizing percentages is to present them on a 100 square grid like this:



In the 100 grid above, square A covers $\frac{1}{100}$ blocks or 1% of the grid.

Time needed
10 minutes

Activity 1:
What percentage is covered?

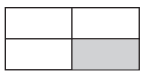
Work alone

1. Fill in the table below to show the same information for blocks, B – E on the grid.

Shaded Area	No of squares	Fraction of the whole	Percentage of the whole
A	1	$\frac{1}{100}$	1%
A+B+C+D+E			

2. Use crayons to colour the grid printed on the previous page as follows:
   Colour 10% red   Colour 17% green   Colour $\frac{23}{100}$ blue
   a. Find out what total % of all the blocks you have coloured
   b. Colour half of what is left green.
   c. What fraction is left uncoloured? What is this as a percentage%?

2. Using percentages



In the above diagram the shaded square is $\frac{1}{4}$ of the big square. To change the fraction to a percentage we can calculate in this way:

$\frac{1}{4}$ of 100 which is the same as

$\frac{1}{4} \times 100$

which is 25, so that's 25%.

And $\frac{3}{4}$ of 100 is unshaded.
So that's 25% × 3 = 75%
So if there were 100 blocks 25 of them would be shaded, and 75 would be unshaded.

Or you can use your $\boxed{\%}$ percentage key on your calculator you enter

[1] [÷] [4] [%] [=] Or if you do not have a % key then press:

[1] [×] [1] [0] [0] [÷] [4] [=]

Stop and Think

How would you change $\frac{3}{4}$ to a % on your calculator? Try it now.

Activity 2:
Calculating percentages

Time needed
30 minutes

Work alone

1. Use what you found out to change these common fractions into percentages without using a calculator.

   a) $\frac{1}{2}$ b) $\frac{1}{5}$ c) $\frac{1}{4}$ d) $\frac{1}{10}$

   e) $\frac{3}{4}$ f) $\frac{4}{20}$ g) $\frac{17}{20}$ e) $\frac{21}{25}$

2. Now use the calculator to change these fractions into percentages:

   a) $\frac{1}{8}$ b) $\frac{3}{40}$ c) $\frac{5}{32}$ d) $\frac{1}{3}$ e) $\frac{5}{11}$

Stop and Think

What did you notice about the answers to 2 d) and e)?
We will discuss these kinds of numbers later on in the unit.

3. Out of 50 tins of jam that were ordered by the Bantwana Bami ECD Centre, 24 had labels. Write this as a percentage.
4. Mrs. Dlamini only had $\frac{14}{25}$ children present in her class today and Mrs. Langa had $\frac{15}{20}$ children present. Which teacher had the higher percentage of children present?
5. 6 minutes of an hour-long TV show is taken up with advertising. What percentage is not used for advertising?

Work with a partner

6. Compare and discuss the different methods you used to solve the above problems.

What have you learned

It is easier to compare the value of fractions by changing them into percentages. So in the example of question 4 above to compare which teacher had a greater percentage of children present you can do a comparison by converting each fraction to a percentage as follows:

$\frac{14}{25} \times \frac{4}{4} = \frac{52}{100} = 52\%$

And

$\frac{15}{20} \times \frac{5}{5} = 75\%$

Time needed
20 minutes

Activity 3:
Writing decimal fractions as percentages

Work Alone

An easy way to change a % to a decimal fraction is to first write the decimal fraction as a common fraction and then as a percentage, like this;

$0{,}45 = \frac{45}{100} = 45\%$

1. Do these in the same way
   a) 0,25 b) 0,32 c) 0,12 d) 0,08 e) 0,42 f) 0.5

   0.075 as a percentage is 7,5% because 0,075 x 100 = 7.5

2. Write these amounts as a percentage
   a) 0,012 b) 0,037 c) 0,125 d) 0,375

Time needed
30 minutes

Activity 4:
Changing percentages into decimals and fractions

Work alone

You can also change a percentage into a decimal, for example:
$10\% = \frac{10}{100} = 0.10$ or $0{,}1$

1. Change these percentages into decimals
   a) 25% b) 47% c) 8,5% d) 1%

   And you can just as well change a percentage into a fraction.
   In this case you would need to simplify your fractions

   e.g. $20\% = \frac{10}{100} = \frac{10 \div 10}{10 \div 10} = \frac{1}{10}$

   or $35\% = \frac{35}{100} = \frac{35 \div 5}{100 \div 5} = \frac{7}{20}$

2. Change these percentages into fractions
   a) 20% b) 40% c) 85% d) 90%
   e) 54% f) 16% g) 32% h) 45%

   Sometimes percentage amounts have fractions attached e.g. 12,5%

   This is one way we can change an amount like 12,5% into a decimal

   $12{,}5\% = \frac{12{,}5}{100} = 0{,}125$

Thabi changes 12.5% into a fraction like this;

$$12.5\% = \frac{12,5}{100} = \frac{12,5 \times 10}{100 \times 10} = \frac{125}{1000} = \frac{125 \div 5}{1000 \div 5} = \frac{25 \div 25}{2000 \div 25} = \frac{1}{8}$$

Bertha does it like this;

$$12,5\% = \frac{12,5}{100} = \frac{25}{200} = \frac{1}{8}$$

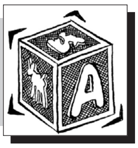
Trainer's Note:
Review learners' answers at this point. Find out if there are any other methods your learners may have used. Have them share these with their colleagues. Give additional practice in similar kinds of activities if necessary.

3. Discuss all of these methods together. Make sure you understand them.
4. Use any method you prefer to change each of these amounts to decimals and then to fractions.

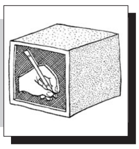
   (i) 37,5%   (ii) 62,3%   (iii) 37,25%   (iv) 1,25%

5. Complete the missing values in this table

Decimal										
Percentage	10%		70%		45%		25%		85%	
Fraction		$\frac{1}{4}$				$\frac{1}{8}$				$\frac{1}{3}$



Time needed
20 minutes



Activity 5:
Facts and figures

Add the work you did on this activity to your portfolio.

Work with a partner
In developing countries, there are more men than women. There are also more women farmers than men in some countries. Look at these figures for example:

Percentage of women farmers (compared to men).
Asia 80%   Africa 80%   Middle East 60%   Latin America 40%

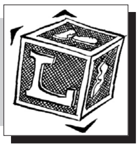
1. Are these statements true or false? Do a calculation to prove your answer.
2. Read this information:
   a. $\frac{4}{5}$ of the farmers in Africa and Asia are women.
   b. Less than $\frac{1}{2}$ the farmers in the Middle East are women.
   c. $\frac{1}{3}$ of the farmers in Latin America are women.
   d. There are twice as many women farmers in Asia than in Latin America.
   e. There are half as many women farmers in Latin America than in Africa.

3. Work out the percentage of male farmers in each place.
4. Express these amounts as fractions.
5. Read this short article then answer the questions that follow:

An African challenge.

Research shows that the better one's education, the better the chances of growing food and improving a country's food security. In Africa it is largely the women responsible for growing food; 80% of African farmers are women, yet $\frac{2}{3}$ of them are illiterate. Worse than that, only about 55% of girl children attend primary schools. "These facts do not bode well for the future", the spokesperson for Food Security at the conference said.

a. What fraction of farmers in Africa are women?
b. What percentage of them are illiterate?
c. What percentage of girls do not go to school?
d. How much is this as a (i) decimal (ii) a fraction?
e. Discuss the conditions described in this article. What steps do you think that governments and other agencies could take to improve the situation?



What have you learned?

Fractions, decimals and percentages are all types of rational numbers that describe proportional relationships and can be used interchangeably. Percentages are particularly useful to use to describe particular social, political or economic conditions or trends.

Time needed
20 minutes

Activity 6:
More about recurring decimals

Work with a partner

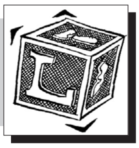
1. Write $\frac{1}{3}$ as a decimal: Write 1 d 3 on your calculator
2. Write $\frac{1}{3}$ as a percentage enter 1 d 3% or 1 × 100 ÷3 =
3. What do you notice?

We call 0,3333333 a recurring decimal and write is as 0,3 We write 0,66666 as 0,6 and

0,27272727272 as 0,27

Copy and complete this table

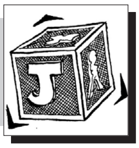
Common fraction	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{4}{9}$
Decimal fraction					
Percentage					



What have you learned?

- Percentages are a commonly used as an efficient way to describe number relationships as proportional shares. Like other rational numbers they describe the relationship between two integers. In this case the one value is always 100 as percentage means out of 100. There are different ways once can use to calculate and converting percentages.

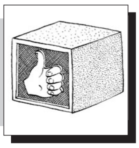
- Of course you can always use a calculator, but you can also apply your number knowledge to do quick percentage calculations in your head or by writing out your calculations. Percentages, decimals and common fractions are different ways of expressing the same value and one can convert from any form of the number to another by using different techniques and of course by using a calculator.



Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about percentages?
b. Write down one or two questions that you still have about percentages.
c. How do you think you will be able to improve your understanding of percentages?
d. How will you use what you learned about percentages in your everyday life and work?

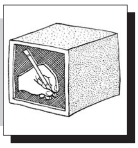


Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Explain the meaning of the term percentage				
2. Name percentage amounts that are both shaded and un-shaded in different square grids				
3. Express percentage amounts as common fractions - and the other way round				
4. Express decimal amounts as percentages - and the other way around				
5. Solve real world problems where you have to calculate percentage amounts				
6. Use the correct keys on your calculator to do percentage calculations and to check your manual calculations of percentages				
7. Investigate and explain the value and meaning of different recurring decimals				



Assignment 4

In any newspaper you will find many afferent examples of percentages being used to report on different facts and figures. Try this out. Buy a newspaper and look through it to find examples of where percentages are used, not only the financial sections, but in general articles. Write a short description to explain the different contexts you found out about. When you are finished compare your findings with a partner. Repeat the exercise a week or two later and see if you find new and different examples. An example of how your record your findings is shown below:

Name/date of newspaper	Page	Topic of article	References to percentages
The Sowetan 21/1/2006	5	School enrolments soar.	The article says that the number of Grade R enrolments this year has increased by 7,2 % from 2005.

UNIT TEN

Working with Money

DICTIONARY:

algorithm – rules for calculation

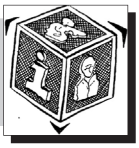
In this unit you will address the following:

Unit Standard 7449
SO2:
Critically analyze the use of mathematical language and relationships in the economy. (Budgeting, banks: interest rates, mortgage, service charges; fuel prices; pensions; inflation; value of the rand and exchange rates.)

SO3:
Critically analyze the use of mathematics in social relations. (Social differentiation: gender, social mobility, race; historical and possible future contexts, e.g. employment equity; apartheid policies.)

SO4:
Critically analyze use of mathematics & mathematical language & relationships in political relations (Income distribution; census; elections; voting; opinion polls.)

To do this you will:
- work with money to calculate discounts, commission, profit and interest;
- work with personal budgets;
- work with exchange rates.

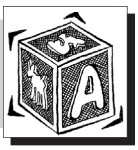


1. Calculating money

Throughout this module we have tried to emphasize that there are many different methods you can use to do calculations with numbers. These methods are often much easier and make more sense than the standard algorithms that most of us were taught at school. So to calculate R52 × 8, you might think of it in some of these different ways:

(R50 × 8) + (R2 × 8)
or as R52 × 2 × 4
or as (R52 × 4) + (R52 × 4)
or as (R52 × 10) – (R52 × 2)

Can you see that in all cases your answer will be the same: R416



Time needed 60 minutes

Activity 1:
Quick calculations

Work alone

1. Find your own quick ways to calculate these amounts
   e.g. R26,50 × 2 = (2 × R26) + (2 × 50c)
   = R52 + R1 = R53

> ***Trainer's Note:***
> *Make time for learners to discuss and share their different methods they may have used here and to evaluate which of them are easiest and to say why.*

a) R12.05 × 25	b) R132,45 × 3	c) R135 × 9	d) R449.25 × 2
c) R44 000 ÷ 2	d) R44 000 ÷ 20	e) R88 000 ÷ 4	f)R88 000 ÷ 40
g) R196 000 ÷ 50	h) R1 960 000 ÷ 500	i) R19 600 000 ÷ 5 000	

2. In this large amount: R56 791 465.85 the 6 stands for six million
   a. Name the value of each of the other digits that make up this amount.
   b. Write the amount out in words.

3. Do these same for these amounts:
   (i) R327 843 415   (ii) R10 843 627.05   (iii) R6 872 400.85   (iv) R19 999 999

Work with a partner

4. Discuss and compare answers. Make any corrections you need to.

Work alone

5. How much is?
   a) $\frac{1}{2}$ of R250 000   b) $\frac{1}{4}$ of R250 000   c) $\frac{3}{4}$ of R250 000
   d) $\frac{7}{10}$ of R20 000 000   e) $\frac{1}{5}$ of R20 000 000   f) $\frac{7}{10}$ of R20 000 000

6. Do calculations to find out which of the amounts below is not the same as $\frac{1}{2}$ of R250 000
   a) $\frac{1}{4}$ of R500 000 b) $\frac{2}{8}$ of R250 0000   c) $\frac{1}{8}$ of R1000 000 d) $\frac{4}{8}$ of R250 000

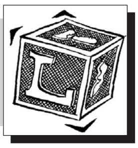
7. Find quick ways to work out the amount of change you will get each time:
   a. You pay for a food bill of R1 638,95 with 9 × R200 notes.
   b. Your ECD centre spends R12 785 on fencing. They pay the company with 130 × R100 notes.
   c. A local church group spends R19 405 for hiring equipment for a community event.
   d. They pay with 500 × R200 notes.

8. Discuss and compare your answers and your calculation methods in 8. with a partner. Evaluate which ways are easiest to follow.

9. Jonas goes shopping for food with his two daughters. When he has bought the goods on his list, his daughters ask him to buy them some apples and some yoghurt. His budget for shopping is R350. The apples and yoghurt will cost R18.65 together. Estimate the cost of the goods Jonas has bought so far.
   a. Has he enough money to pay for the yoghurt and apples?

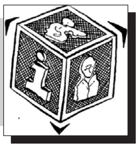
2 litres of oil R18.60	4 litres of milk R24	2kg of sugar R17.50
4kg meat R87.50	5kg of maize meal R26	1kg of coffee R15.40
1 packet of tea R14.25	4kg of bananas R17.40	1kg of onions R7.75

b. Compare your estimations with a partner. Discuss some of the quick ways you might have used to estimate your total. Then use a calculator to find the actual amounts and to check how close your estimations were.



What have you learned?

You can use your number skills to do quick calculations with money, without using a calculator. Rounding off and compensation methods or looking for cent amounts that add up to whole rand amounts are some strategies you might have used. Estimation is something we do almost every day and especially in the context of money so it helps to be able to practise this skill whenever you can!



2. Interest

When you borrow money or buy goods in installments over a period of time, you usually have to pay interest on the cost of the item you are buying. Here's a simple example:

Mr. Latifa bought a radio for R480 on a 12 month credit basis. He had to pay simple interest of 10%. How much more was this?

There are different ways to work this out.

Method 1:

$$\begin{aligned}(100\% + 10\%) \text{ of R480,00} &= 110\% \text{ of R480}\\ &= \tfrac{110}{100} \times \text{R480}\\ &= \tfrac{11}{10} \times \text{R480}\\ &= 11 \times \text{R48}\\ &= \text{R528}\end{aligned}$$

Method 2:

$$\begin{aligned}10\% \text{ of R180,00} &= \tfrac{10}{100} \times \text{R480,00}\\ &= \text{R48,00 (this is the interest)}\\ \text{Total amount} &= \text{R48,00} + \text{R480} = \text{R528,00}\end{aligned}$$

Time needed
30 minutes

Activity 2:
Interest and discount

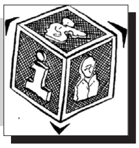
Work in a group

1. Discuss and compare the different methods above in a group.
2. Use either method above to find out how much you will pay altogether for a TV set from a shop that costs R2 250. Interest over 6 months is 12%.
   a. If you split this amount into six monthly installments, what will you have to pay each month? Round your answers off to the nearest 10c.
   b. Use a calculator to check your answers. There may be slight differences because of rounding off.

3. Copy and complete:
   a. If you pay 10% more you pay $\frac{10}{100}$ or $\frac{1}{10}$ more
   b. If you pay 20% more you pay .... or ...... more.
   c. If you pay 25% more you pay .... or .... more.
   d. If you pay 40% more you pay .... or .... more.
   e. If you pay 50% more you pay .... or .... more.
   f. If you pay 75% more you pay .... or .... more.
   g. If you pay 60% more you pay .... or .... more.

4. Find quick ways to calculate the interest on these invested amounts:

Invested Amount	**R10 000**	**R50 000**	**R100 000**	**R100 000**	**R100 000**	**R50 000**	**R5 000 00**	**R50 000**
% Interest	10%	20%	20%	5%	25%	10%	5%	4%
Amount	R1 000							
Total	R11 000							



3. Discounts

Discounts are worked out in the opposite way. If you buy goods at discounted prices, you get a certain percentage off. For example how much will you pay for a radio that costs R480 where the discounted price is less 20%?

Method 1

(100% - 20%) of R480,00 = 80% of R480
= $\frac{80}{100} \times$ R480,00
= $8 \times$ R48 ← Can you explain this step?
= R384,00

Method 2

20% of R480, 00 = $\frac{80}{100} \times$ R480, 00
= 2×48 ← Can you explain this step?
= 96
= R480 – 96
= R384,00

Time needed 20 minutes

Activity 3:
What's the sale price?

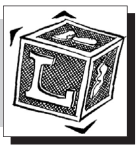
Work alone

1. The principal of Bantwana Bami, Mrs. Maseko, went shopping for items she needed at the Centre. Use either method shown above to find out what the sale price is for these items. Do not use a calculator!
   a. A set of playing blocks; was R690; now less 10%
   b. A set of 5 easels were R750 a set; now less 15%
   c. A set of plastic shape pieces; were R286; now less 20%
   d. Sandpit toys were; R196 now 30%
   e. A set of powder paints; were R360 for 12; now less 25%

f. A set of plastic tables and chairs, was R256; now less $12\frac{1}{2}\%$
g. A bag of plastic counters; were R78.80; now less 40%

2. Use a calculator to check your answers.

Submit your answers, showing your methods of calculating and all the steps you followed, to form part of your portfolio assessment.



What have you learned?

Interest is the amount of money you either pay on money borrowed or earn on money invested. Discounts are based on a % if the usual sale price. There are different ways you can calculate interest amounts or discount prices. For example Thembi found the discounted price of the sandpit toys in Question 1d) by thinking about it like this:

R196 = 100%
10% off = R19.60
30% off = 3 × R19.60 = R58.80
So sale price = R196 – R58.80 = R137.20

And Tim thought about it like this:

"If there is 30% off then the sale price is 70% of the original price"

$\frac{70}{100} \times \text{R}196.00$
$= \frac{7}{100} \times \text{R}196.00$
$= 7 \times 1960 = 13\,720$
$= \text{R}137.20$

4. Profit

The cost price is the amount you paid for the item. The selling price is the amount you sell the item for. When you sell an item for more than you bought it, you make a profit

Your selling price is your cost price plus your profit (or your mark up). You can write this as a formula. Remember SP (selling price) = CP (cost price)+ P(profit). Your profit is your selling price minus your cost price. So the formula is

P = SP – CP

Your cost price is your selling price minus your profit. So the formula is

CP = SP – P

Time needed
20 minutes

Activity 4:
Calculating profits

Work alone

Ma Serobe runs a Spaza next door to the Bantwana Bami ECD Centre. She marks up each item by a certain % to make a profit on her sales.

1. Copy and complete the table below to calculate the profit and selling price of each item listed in the table. Round your answers to the nearest 10c, where necessary.

	Item	Cost price	Profit markup	Profit amount	Selling price P
a)	E.g. 10 Packet of biscuits	R30.00	20%	R6	R36.00
b)	100 Crate of cold drinks	R440.80	15%		
c)	10 kg Bananas	R35.70	25%		
d)	4 pockets of oranges	R36.40	12%		
e)	12 kg potatoes	R62.40	25%		

2. A new mining company predicted that their annual profit would be R2m. Their profit figures showed 20% less than this. How much is this?
3. The annual profits of a large international manufacturing company are R45 678 950 for the current year. This is 25% more than last year.
   a. How much were their profits last year?
   b. Next year they expect an increase in profits of at least 8% more than this year. How much is this?
   c. How much more is this than their profits for last year?

Work with a partner

4. Use a calculator to check your answers. Discuss the different methods you used. Evaluate which methods are easier to follow. Correct any mistakes you made.

5. Commission

Commission is a percentage of sales, paid to the person who made the sales. Commission can also be calculated based on the overall profit of the company.

Time needed
20 minutes

Activity 5:
Calculating commission

Work alone

Pinkie's husband is a travelling salesman. He only earns a small salary but he earns most of his money on commission. He gets 20% of all sales, so if he sells a lot, he can earn a lot. This means if he sells R2 000 worth of goods he makes $\frac{1}{5}$ of R2 000 as commission. That is R400.

1. Calculate the commission he would have earned on the following sales. Find quick ways to do this without using a calculator!

Value of sales	2 000	20 000	100 000	10 000	25 000	250 000
Commission earned @20%	R400					

Work with a partner

2. Discuss and compare answers and methods of calculating with a partner. Make any corrections you need to.

6. Simple interest

When we open a savings account at a bank we can earn money on our investment.
This is also called interest, interest we earn rather than interest we are charged!
The interest is a % of the amount of money invested. Different kinds of accounts offer different interest rates. Interest rates may be fixed for a given period or they can change depending on how the bank rates go up or down. Banks make money by lending money to its clients at a higher rate of interest than it pays its clients for the money they invest.

Activity 6:
Interest earned

Time needed
20 minutes

Work alone

Let's take an example of a savings account that offers 8% interest per year, written as 8% p.a. (per annum). In the table below the first row tells you the different amounts to be invested at a rate of 8%.

1. Calculate how much interest this is over a year and write your answers in the second row.
2. In the third row calculate how much interest this is per month (rounded to the nearest 5c)
3. In the fourth row calculate how much interest could be earned at this rate after 3 years.

Amount Invested	R5 000	R3 500	R7 000	R15 000	R1 750	R17 500
Interest earned at 8% p.a.						
Monthly interest						
Interest after three years						

4. In your own words describe a formula that describes how simple interest is calculated.

What have you learned?

In the third row you probably multiplied the interest earned in a year by the number of years. So the formula for calculating simple interest can be written like this:

$$I = \frac{P \times r \times t}{100}$$

P = money invested
r = interest rate per year
t = number of years

Activity 7: More interest

Time needed 30 minutes

Work alone

1. Use the formula to calculate how much interest you can earn on
   a. R5 000 invested for 12 years at 12%.
   b. R10 000 for 7 years @ 14% interest.
   c. R13 300 at 12,5% over 6 months.

2. If you invested an amount of R120 a month for one year and your investment was worth R792 at the end of the year, what % interest did you earn?

Submit your answers to this activity showing all your calculations.

What have you learned?

Percentages are used very often in the context of money. In these activities you worked with percentages to calculate the interest charged and earned and in relation to commission and on profit margins. There are just some examples. You will come across the use of % many more times than this in your daily life so it is useful to know how to calculate % both manually and using a calculator. You can use percentages to compare prices and costs and to make informed financial decisions. As you already know both fractions, decimals and percentages are different kinds of rational numbers that can be used to describe the same relationships so it is useful to know for example that 50% is the same as $\frac{1}{2}$ or 25% is the same as $\frac{1}{4}$.

Time needed
30 minutes

Activity 8:
Rate of pay

Work alone

This advertisement gives information on the rate of pay for two different jobs. Rate here means the amount of money you can earn in a given time period. Do not use a calculator, but rather try to find quick and clever ways to do these calculations on your own:

> **Jobs Available call now!**
>
> *Tele – Sales Work from home Part-time, own phone essential,*
> *R120,00 per hour.*
> *Office temp needed for holidays R75 per hour 40 hours a week}*

1. Thuli is hired as the office temp. Calculate her weekly wages if she works for 13 weeks. She has to pay 25% of this amount on tax. How much pay does she take home?
2. Jane is employed as the tele-sales assistant. How much does she earn?
   a) for a 25 hour week. b) for 15, 25-hour weeks?

3. Rhadya opens a flower shop. She employs a part-time assistant Lilly, who works for 3 hours every morning from Monday to Friday. Rose pays her R27.50 per hour.
   a. How much money does Lilly earn weekly?
   b. She has to pay 25% of this amount in tax. What is her take home pay each week?
   c. The shop makes R485 on Monday, R920 on Tuesday, R1 010 on Wednesday, R1 685 on Friday, and R1 850 on Saturday. Rhadya has to pay Lilly her salary and R820 to cover other expenses. The rest represents her profits. How much is this?

Work with a partner

4. Discuss and compare calculation methods with a partner. Use a calculator to check your solutions. Make any corrections you need to. Evaluate which methods were easiest to follow and why.

7. Working overtime

Usually people work for 8 hours a day, including a lunch break for an hour. If employees are expected to work longer than this, they can ask for overtime pay. Higher hourly rates are usually paid for overtime work. As a general rule:

- Working on evenings and Saturday afternoon after 1p.m. = $1\frac{1}{2}$ times the hourly rate.
- Working on Sundays and public holidays = 2 times the hourly rate.

Activity 9:
Overtime pay

Time needed 15 minutes

Work alone

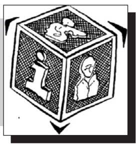
Work out these problems on overtime pay. Do not use a calculator!

1. Karabo earns R63.50 an hour as a laboratory assistant. She works four extra hours overtime on a Saturday morning to help her supervisor analyse some samples. How much will she earn for this time?
2. Mandla is a trainee hairdresser. He earns R33 per hour in normal time and earns the overtime rate when he works out of normal hours. Calculate what he earns for:
   a) 32 hours of weekdays work b) 14 hours of overtime.
3. Discuss and compare your calculation methods and your answers with a partner. You can use a calculator to check your answers. Make any corrections you need to.

Trainer's Note:
Have learners do some of their own research on current salaries offered in job advertisements and to calculate for example what a monthly salary of R7 000 translates into as an hourly rate for a 40 hr week. If overtime rate of $\frac{1}{4}$ *times the hourly rate is payable, how much would this be for e.g. 15 hours of overtime. If tax is charged at 30% what is the take home pay? Also ask them to look for examples of jobs advertised where only commission is earned, or a minimal basic salary and commission, and to project what kind of earnings can be made.*

What have you learned?

Salaries are paid according to different rates, which might be monthly, weekly, daily or even hourly. It is useful to compare these when applying for a job and to see, for example, if a weekly rate of x amount works out at roughly the same or perhaps even better than a monthly rate of y, to evaluate which is the better option.



8. Personal budgeting

Working out how much money we have to spend and save is called budgeting. To budget you need to know your income and your expenses. Some people receive an income weekly, others monthly, whiles others daily. Some expenses occur monthly like rent, food transport, school fees, while others less frequently like clothes, equipment, appliance repairs. It is always good to have a reserve in your budget to cater for those unexpected expenses that occur when you least expect them.

Here is a simple way to draw up a budget for yourself. This will help you to know how to plan your spending and to see if you are over or under budget for any essential items and if you can afford those little extras that you may want but are not sure if you can afford.

Step One

Calculate your monthly income and include contributions from other members of the family who may be contributing to your monthly expenses.

Step Two

Calculate your weekly expenses and multiply these by four to give you one month. In some cases like school fees the expense will be monthly, rather than weekly. Record your expenses in a table like Thuli did below.

Income			**R5 250**
Weekly Expenses	**Cost**	**Monthly Expenses x 4**	
Groceries	R220	R 880	
Travel	R180	R 720	
School fees		R 250	
Bon repayments		R1 350	
Savings account		R 400	
Add other categories			
Total			
Subtract total expenses from income to see of you are spending more than, less than or as much as your monthly income!			

Time needed
20 minutes

Activity 10:
Personal budgeting

Work alone

1. Use a calculator find what percentage of Thuli's total income is she spending one each category of expenditure?
2. Draw up a budget like this for yourselves to help you track your own spending. Maybe you are spending more than you are earning? Maybe you are even saving money each month you could invest?

When the government works with budgets they work with figures that run into millions and billions of Rands. In South Africa a billion is equivalent to a thousand million. In some countries it is a million million! Look at this place value chart to help you visualise how much a billion is and to find out how we write and say numbers in the billions.

Billions (Thousand Millions)	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
1	0	0	0	0	0	0	0	0	0
7	0	0	1	5	0	0	0	0	4

Stop and Think

In power notation we write a billion as 1^9.
Do you see why this is so?
The number in the second row is read as: seven billion, one million, five hundred thousand and four. Can you see why?

Time needed
40 minutes

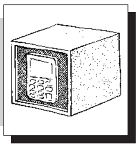
Activity 11:
Government budgets

Work with a partner

The following table gives you information about budget allocations for social services, including education from the 2004-5 national budget. (Figures given in billions of Rands). Read the information carefully.

	Education	Health	Social Security	Housing	Community Development	Total in Rands
Allocated Amount in Rb	75.9	42.6	59.9	6.3	12	
						Total as a % of national budget
% of total national budget	19,6%	11%	15,5%	1,6%	3,1%	

2004-5 National Budget (Social Services, incl. Education)

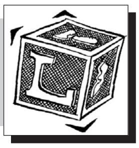


1. Write each budget amount in words, using the figures given in the table on the previous page. eg. 75.9b = seventy five billion, nine hundred thousand.
2. Calculate the total budget allocation from the national budget for social services, including education.
3. Find what % of the overall budget that this represents.
4. The budget allocation for economic services was R196.7b and for protection services was R63.2b? How much more than these was the allocation for Social Services?

We could calculate the % or fraction of the total budget for Social Services that was spent on education like this:

$$\frac{\text{R75.9b}}{\text{Total budget you calculated in 2}} \times 100$$

5. Use this method to find the % percentage allocated to each sector under the social services budget, including education.
6. Discuss and compare your methods and answers with other colleagues working on the same tasks.



What Have You Learned?

Budgets are part of our everyday lives. Whether they are our personal budgets or government or provincial budgets, in one way or another we all think about the amount of money that we have available to buy things, and to access services available to us from government budgets. Keeping your own personal budget can help to keep track of your spending and to know whether you are living within or above your means.

Government budgets run into millions and billions of Rands and it is interesting to know what the budget allocations for different services are from year to year. We can only really evaluate, for example, whether enough money is being spent on education if we compare this expenditure on other important services that government budgets provide for. We can also analyse if the education budget has increased or decreased over a period of time.

9. Exchange Rates

Everyday you find a section in the newspaper that gives you the exchange rate for some of the currencies that are relevant to the South African situation. Every country has its own currency. The value of one currency in relation to another is called the exchange rate. The rate is not fixed but changes all the time. When you travel to another country or trade with another country you have to pay and charge for things in their currency.

You can write the exchange rate as a ratio:

- The Rand to Zimbabwean dollar exchange rate is 0,12:1, so you have to pay R0,12 for 1 Zimbabwean dollar.
- The Rand to British Pound exchange rate was R11,70:1, and then you would have to pay R11,70 for one British Pound.
- The Rand to US dollar exchange rate was 6,86:1, and then you would have to pay R6,68 for one US dollar.
- If the Rand to the Euro exchange rate is 7,80:1 on a particular day that means you have to pay R7,80 for 1 euro.

Activity 12:
Working with exchange rates

Time needed
40 minutes

Work with a partner

1. Which currency used in the examples above is worth less than the Rand?
2. Which country will be the most expensive to travel to? Why?
3. Thandi wants to buy a dress costing 45 British pounds when she is in London. First estimate, and then calculate how much this will cost her in Rands.
4. The euro is the common currency used by members of the common market in Europe.
5. List all the countries you know that belong to the common market.
6. How much will you have to pay for these amounts of Euros at an exchange rate of 7.80:1? Look for links to find quick answers.

No of Euros	1	10	5	50	100	1 000	500	250	750	1 500	15 000
Cost in Rands	7,80										

7. Mrs. Maseko orders a book about natural cures for children's illnesses on the Internet. The book costs $25,50 (US dollars.) The postage costs another $9.25. How much will she have to pay for the book in Rands?
8. A grain farmer in Limpopo can sell his grain to Zimbabwe for 6 000 Zimbabwean dollars a bag, How much is this in Rands?
9. Bheki comes back from Zimbabwe with 15 000 Zimbabwean dollars. How many Rands is this worth?
10. A South African artist sells a sculpture to a New York museum for R84 000. He is paid in US dollars. How many US dollars does he receive?
11. Discuss and compare answers in your group. Make any corrections you need to.

What have you learned?

Exchange rates are useful to know about. When you travel to another country or do trade with another country you need to know what the value of the Rands you use are and what equivalent amount they can buy in other currencies. Almost all of us have been in a position where we have had to think about this at one time or another. With the trend in globalization and the increased trade and contact with other countries, knowing about the exchange rate becomes more critical.

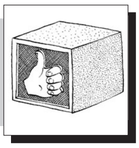
The exchange rate is affected by many different economic and political conditions. If there is political unrest in a country or a poor level of economic activity, then it is likely that the value of that country's currency is going to be far lower than that of established and more stable countries such as countries in EU, America or Britain.

The health of a country's economy is often measured in relation to its exchange rate with the US dollar, the Euro or the British Pound. So one might argue that the lower the exchange rate, the stronger the rand, and the more healthy the SA economy is. However if our economy depends heavily on the export market, this is a problem because traders will get less for their Rands than when the exchange rate is higher. For this reason there are constant debates in the media as to whether it is a good or bad thing that the Rand has strengthened significantly against these currencies over the last few years.

Journal Reflection

Think about what you have learned. Write down all your thoughts, ideas and questions about your learning in your journal. Use these questions to guide you:

a. What did you learn from this unit about working with money?
b. How do you think you will be able to improve your skills of working with money?
c. Write down one or two questions that you still have about working with money.
d. How will you use what you learned about money in your everyday life and work?

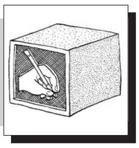


Self-assessment Checklist

Reflect on the outcomes that were set for this unit. Think about what you know, what you can do and how you can use what you have learned. Use the key in the table and tick the column next to each outcome to show how well you think you can do these things now.

I can:

Tick ✓ as follows: 4=Very well 3=Well 2=Fairly well 1=Not well.	4	3	2	1
1. Work with money to calculate discounts, commission, profit and interest				
2. Work with personal budgets				
3. Work with exchange rates				

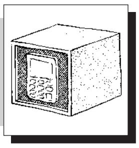


Assignment 5

The exchange rate changes from day to day. The rates we have used in this activity may be different from the one you find in the paper. If you look up these rates in the newspaper today, they will be different to the ones we have given you here.

1. Find information about today's exchange rate for these currencies:

   The Zimbabwean $; the Botswana Pula; the US $; the Euro; the British Pound



   a. Write these values as a ratio in relation to the Rand e.g. 7:1 means $7 dollars to R1.
   b. Find out what R5 000 would buy you in each of the different currencies.
   c. What will it cost you to buy 5 000 English Pounds?
   d. What will it cost you to buy 5 000 US dollars?
   e. What will it cost you to buy 5 000 Botswana Pula?
   f. How much will you pay for 50 000 Zimbabwean dollars?
   g. How much will you pay for 5 000 000 Zimbabwean dollars?

2. Compare your answer to f) with the cost of buying 5 000 000 English Pounds.

3. You have the option of ordering R20 000 worth of educational equipment for your ECD Centre from Europe, the US and England. Work with the exchange rates you have found out about to decide which country it would be best to purchase your goods from.